Bond Market Exposures to Macroeconomic and Monetary Policy Risks

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The paper estimates a model that allows for shifts in the aggressiveness of monetary policy and time variation in the distribution of macroeconomic shocks. These model features induce variations in the cyclical properties of inflation and the riskiness of bonds. The estimation identifies inflation as procyclical from the late 1990s, when the economy shifted toward aggressive monetary policy and experienced procyclical macroeconomics shocks. Since bonds hedge stock market risks when inflation is procylical, the stock-bond return correlation turned negative in the late 1990s. The risks of encountering countercyclical inflation in the future could lead to an upward-sloping yield curve, like in the data. (*JEL* E32, E42, E43, E44, E52, G12)

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In the current macroeconomic environment, several stylized bond market facts are different from those in the previous decades: Inflation risk premium is low and possibly negative; and the correlation between U.S. Treasury bond returns and stock returns, while positive in the 1980s, has turned negative in the last decade.[1,2] There is an understanding in the literature that these new facts can be reconciled in models that allow for exogenous changes in the cyclical properties of inflation, for example, Burkhardt and Hasseltoft (2012) and David and Veronesi (2013). When inflation is procyclical nominal bonds are safe and provide a hedge. Since nominal bonds behave similar to real bonds

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[1] See Fleckenstein, Longstaff, and Lustig (2015) and Chen, Engstrom, and Grishchenko (2016).

[2] See Baele, Bekaert, and Inghelbrecht (2010), Campbell, Pflueger, and Viceira (2015), Campbell, Sunderam, and Viceira (2016), and David and Veronesi (2013).

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in this environment, these models imply negative stock-bond return correlation, negative inflation risk premium, and a downward-sloping nominal yield curve. However, in the data, the nominal yield curve still slopes up during the same periods in which the stock-bond return correlation and inflation risk premium are negative. This recent evidence is interesting because it shows the limitations of the existing approaches and highlights the importance of understanding the sources of inflation and bond market risks and how they change over time.

This paper provides an economic mechanism underlying the inflation dynamics and bond markets by introducing three new elements in a consumption-based asset pricing model: (1) a monetary policy rule with time-varying inflation target, (2) shifts in the strength with which the Federal Reserve steers actual inflation toward the inflation target, and (3) shifts in the covariance of inflation target and real consumption growth shocks.[3] The first extension leads to "endogenous" inflation, and the nominal assets inherit the properties of monetary policy. The second and third extensions induce variations in the cyclical properties of inflation, and these lead to the risk premium and the correlation between stock and bond returns switching signs. Agents, who favor early resolution of uncertainty, are aware of the possibility of encountering countercyclical inflation in the future due to changes in the aggressiveness of monetary policy and the distribution of macroeconomic shocks. Consequently, they demand compensations for holding nominal bonds that might be exposed to future inflation risks: risks and compensations are greater for longer-term bonds resulting in an upward-sloping nominal yield curve.

The model features three distinct economic regimes: (1) the CA regime occurs when the conditional covariance between inflation target and real consumption growth is negative (*Countercyclical* macroeconomic shocks) and the Federal Reserve increases interest rates more than one-for-one with inflation (*Active* monetary policy); (2) the CP regime occurs when macroeconomic shocks are *Countercyclical* and the Federal Reserve raises interest rates less than one-for-one with inflation (*Passive* monetary policy); and (3) the PA regime occurs when the conditional covariance between inflation target and real consumption growth is positive (*Procyclical* macroeconomic shocks) and the monetary policy is *Active*.

To quantify the regime risks and see if the proposed economic regimes line up with the existing literature, I estimate the model with Bayesian techniques using monthly information from asset prices (aggregate stock market and nominal Treasury yield curve) and macrovariables (consumption growth, CPI inflation) that range across the 1963-2014 period. Through the estimation of the model, I investigate the role of monetary policy and macroeconomic shocks played in triggering changes in the inflation dynamics and, ultimately, in the bond market.

[3] The model follows the long-run risk literature on the real side of the economy, and extends the previous literature to include the nominal sector and changing economic regimes.

The estimation of the model delivers three important empirical findings. First, the model supports the idea that the U.S. economy was subject to *occasional* regime switches: The CP regime was prevalent until the early 1980s; the economy switched to the CA regime after the appointment of Paul Volcker as Chairman of the Federal Reserve; and it switched to the PA regime in the late 1990s and largely remained in that regime throughout the sample. The historical paths of the monetary policy stance are consistent with the empirical monetary literature.[4] According to the estimated transition matrix, the unconditional regime probabilities for the CA, CP, and PA regimes are 0.35, 0.33, and 0.32, respectively. The unconditional probability of staying in the active monetary policy regime, as indicated by the sum of the probability of the CA and PA regimes, is around 0.67, twice as large as that of the passive monetary policy regime, that is, the CP regime. I employ a battery of robustness checks and find that my results are maintained across various alternative specifications.

Second, the model accounts for significant changes in the inflation dynamics observed in the data. The estimation finds that inflation has become procyclical and less risky as the economy shifted toward an active monetary policy and experienced procyclical macroeconomic shocks. As the economy shifted from the CP regime to the CA regime and to the PA regime, the variance and persistence of inflation decreased substantially. Nevertheless, inflation risks are substantial in the model because the unconditional probability of experiencing countercyclical macroeconomic shocks, as indicated by the sum of the probability of the CA and CP regimes, is 0.68.

Third, the model finds that each regime carries distinctly different inflation risks, and uncertainty about movements across the regimes poses additional risks to bond markets. To understand the properties of regime risks, I conduct two sets of exercises: one in which the regimes are fixed and the other in which regime switching is allowed in the economy. I first characterize each regime risk in a fixed-regime economy, and, subsequently, by allowing regime switching, I isolate the effect of expectations on asset prices. In a fixed-regime economy, agents dislike the CP regime since there is a large shock to inflation target that comes with low consumption growth and the Federal Reserve does not react aggressively enough to it. Inflation risks are significant in this environment. Note that the CP regime is the extreme version of the economy as described in Piazzesi and Schneider (2006), Wachter (2006), Eraker (2008), and Bansal and Shaliastovich (2013), who assume inflation is countercyclical and risky. Their intuitions carry over: The implied risk premium, stock-bond return correlation, and the slope of the yield curve are positive and largest in magnitude among all regimes. The implications of the CA regime are qualitatively similar to those of the CP regime, but implied inflation risks are smaller in magnitude. On the other

[4] Active monetary policy dominates most of the sample after the early 1980s. The paths for monetary policy are broadly consistent with those found in Ang et al. (2011), Baele et al. (2015), Bikbov and Chernov (2013), Bianchi (2012), Clarida, Gali, and Gertler (2000), and Coibon and Gorodnichenko (2011).

hand, in the PA regime inflation becomes procyclical and nominal bonds are hedges and safe. In this regime, the implied risk premium and the stock-bond return correlation are negative, and the nominal yield curve slopes downward in the PA regime.

Once regime switching is allowed, the model is able to generate an upward-sloping nominal yield curve, while maintaining negative bond risk premium and stock-bond return correlation in the PA regime. The yield curve reflects the covariance between the pricing kernel and the bond return over the entire holding period. When we look at the long end of the yield curve, we are effectively averaging over the different regimes since all of them are likely to occur during the next years. This includes the CP regime which is characterized with both higher inflation level and uncertainty. In comparison, a one-period bond risk-premium is determined by the conditional covariance between the pricing kernel and the bond return over the next period, which weights the current regime much more heavily.[5] An analogous explanation holds for the (one-period) conditional stock-bond return correlation. The key takeaway is that regime uncertainty can go a long way in modifying equilibrium outcomes and is a quantitatively very important risk factor in the bond market.

My work is related to a number of recent papers that study the changes in the stock-bond return correlation. Baele, Bekaert, and Inghelbrecht (2010) utilize a dynamic factor model in which stock and bond returns depend on a number of economic state variables, for example, macroeconomic, volatility, and liquidity factors. The authors attribute the changes in the stock-bond return correlation to liquidity factors. Campbell, Sunderam, and Viceira (2016) embed time-varying stock-bond return covariance in a quadratic term-structure model and argue that the root cause is changes in nominal risks in bond markets. Relative to reduced-form studies, my work builds on a consumption-based equilibrium model with monetary policy to identify the driving forces behind the changes in the stock-bond return correlation.

The works closest to my paper are those of Burkhardt and Hasseltoft (2012), Campbell, Pflueger, and Viceira (2015), and David and Veronesi (2013). Burkhardt and Hasseltoft (2012) find an inverse relation between stock-bond return correlations and correlations of growth and inflation. Burkhardt and Hasseltoft (2012) rationalize their findings in a consumption-based asset pricing model with regime switching (in the conditional dynamics of macroeconomic fundamentals) calibrated to data on fundamentals and asset returns. Campbell, Pflueger, and Viceira (2015) examine the role of monetary policy in nominal bond risks using a New Keynesian model. Using macroeconomic fundamentals and asset prices, Campbell, Pflueger, and Viceira (2015) calibrate the model separately over three different subsamples. From the simulation analysis, the authors claim that the change in monetary policy parameters is the main driver

5 I thank an anonymous referee for these helpful comments.

of bond risks. David and Veronesi (2013) estimate an equilibrium model of learning about inflation news and show that variations in market participants' beliefs about inflation regimes strongly affects the direction of stock-bond return correlation.

My paper is distinct from their works along three important dimensions. First, the structural changes in the economy are identified from macroeconomic fundamentals and asset prices without imposing assumptions, for example, known break points, like in Burkhardt and Hasseltoft (2012) and Campbell, Pflueger, and Viceira (2015). Second, I explicitly account for the role of market participants' beliefs about regime switches in inflation and bond prices. I find strong empirical evidence in the data that the anticipation of moving across regimes is one of the key risk factors priced in the bond market. For example, ignoring the role of beliefs overstates (understates) the implications of a passive (active) monetary policy regime or countercyclical (procyclical) macroeconomic shock regime because the risk properties of alternative regimes are not incorporated. Campbell, Pflueger, and Viceira (2015) do not allow for a beliefs channel to operate. Third, my model exhibits a richer structure than that of David and Veronesi (2013). By accounting for time variations in the covariance matrix of macroeconomic shocks and in monetary policy parameters, I am able to provide extensive descriptions of the bond market transmission mechanism of monetary policy and macroeconomic shocks. In this regard, my model complements the work of Burkhardt and Hasseltoft (2012), Campbell, Pflueger, and Viceira (2015), and David and Veronesi (2013).[6]

By investigating the time variation in the stance of monetary policy, my work also contributes to the monetary policy literature, for example, Baele et al. (2015), Bianchi (2012), Clarida, Gali, and Gertler (2000), Coibon and Gorodnichenko (2011), Davig and Doh (2014), Lubik and Schorfheide (2004), Schorfheide (2005), and Sims and Zha (2006).[7] While most of these papers study the impact of changes in monetary policy on macroeconomic aggregates, the papers of Ang et al. (2011), Bikbov and Chernov (2013), Shaliastovich and Yamarthy (2015), and Ireland (2015) focus on their bond market implications (using reduced-form modeling frameworks). My work distinguishes itself from these papers, since I focus on a fully specified economic model and characterize time-varying bond market exposures to monetary policy risks.

In terms of modeling the term structure with recursive preferences, this paper is closely related to those of Gallmeyer et al. (2007), Eraker (2008), Bansal and Shaliastovich (2013), Le and Singleton (2010), Doh (2012), Creal and Wu (2016), and Piazzesi and Schneider (2006), who work in an endowment economy setting, and van Binsbergen et al. (2012) and Kung (2015), who study a production-based economy. While van Binsbergen et al. (2012) and Kung

[6] Ermolov (2015) considers the stock-bond return correlation in a model with exogenous consumption and inflation dynamics. Ermolov's work came out after the first version of my paper.

[7] Note that I am including those that explicitly account for changes in monetary policy.

(2015) allow for capital and labor supply and analyze their interaction with the yield curve, which are ignored in my analysis, they do not allow for time variation in monetary policy stance, which is a key risk factor in my analysis.

1. Empirical Evidence on Structural Changes

In this section, I empirically document changes in the cyclical properties of inflation, the Treasury yield curve, and the correlation between bond and stock returns.

A recurrent theme of macrofinance term structure models that underlies risk premiums is that inflation uncertainty makes nominal bonds risky.[8] A common approach, supported by empirical evidence, is to assume that inflation is countercyclical. Inflation erodes the value of nominal bonds precisely at times during which consumption growth is low (or marginal utility is high). In this environment, investors demand compensation for holding nominal assets exposed to inflation risk. Since longer-term bonds require greater compensation for this inflation risk, this implies that the nominal yield curve ought to slope up, like in Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2013). Note that since large inflation shocks always come with low real growth, real stocks are also exposed to inflation risk. Therefore, the implied stock-bond return correlation is positive.

This intuition hinges on the empirical correlation between inflation and consumption growth. This correlation, however, does not appear to be robust over different subsamples. To see this, I compute rolling correlation estimates between various measures of inflation and growth over the rolling windows of five years, which are provided in Figure A-2. The correlation has become positive over the last 15 years. The sign-switching pattern tells us something about low-frequency variation in inflation and growth dynamics. To investigate this issue formally, I estimate the state-space model illustrated in Piazzesi and Schneider (2006).[9] Specifically, I assume that the vector of inflation (π_t) and consumption growth (Δc_t) has the following regime-switching state-space representation:

$$z_t = \mu(S_t) + x_{t-1} + \varepsilon_t, \quad z_t = [\pi_t, \Delta c_t]'$$
$$x_t = \phi(S_t) x_{t-1} + \phi(S_t) K(S_t) \varepsilon_t, \quad \varepsilon_t \sim N(0, \Omega(S_t)). \tag{1}$$

The state vector x_t is two dimensional and contains expected inflation and expected consumption growth; ϕ is the 2×2 autoregressive matrix; K is the

[8] Macrofinance term structure models refer to models in which the pricing kernel directly comes from a utility-maximization problem. Gürkaynak and Wright (2012) provide a nice overview of macrofinance term structure models.

[9] Piazzesi and Schneider (2006) empirically document the negative correlation between inflation and consumption growth in the U.S. data. I use their model to show that the correlation changes over time. Other model specifications lead to similar results.

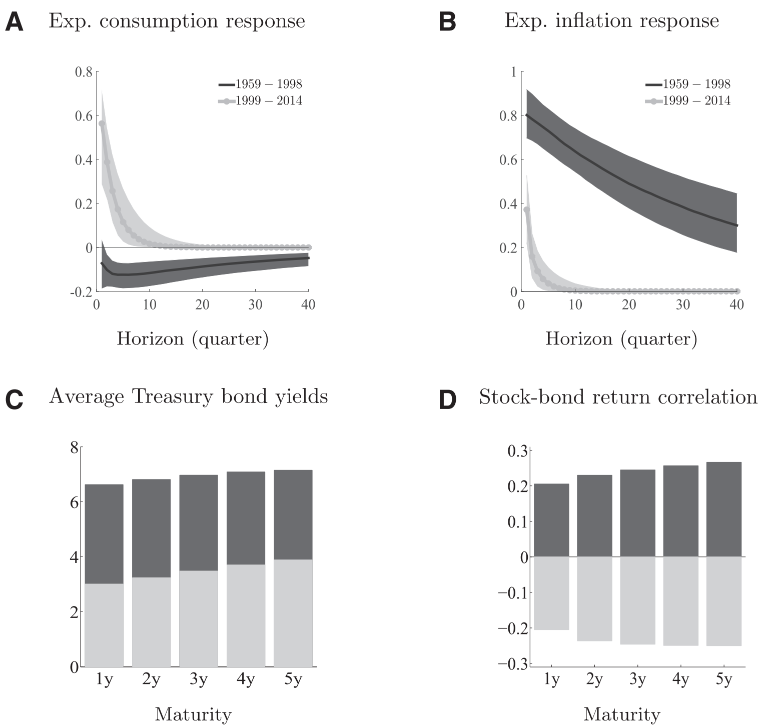

Figure 1
Macroeconomic Fundamentals and Treasury Yield Curve
In (A) and (B) the black (light-gray circled) line represents posterior median expected consumption and inflation reactions to one-percentage-point surprises in inflation from 1959 to 1998 (1999-2014). The black- and light-gray-shaded areas correspond to 60% credible intervals. In (C) the black (light-gray) bars represents the averages of the U.S. Treasury bond yields (annualized) for maturities of 1-5 years from 1959 to 1998 (1999-2014). In (D) the black (light-gray) bars represent the correlation between stock market returns and bond returns for a one-month holding period for maturities of 1-5 years from 1959 to 1998 (1999-2014).

2×2 gain matrix; and S_t denotes the regime indicator variable $S_t \in \{1,2\}$. Using Bayesian methods, I estimate this system with quarterly consumption and CPI inflation data that span 1959 to 2014. Details (about priors and posterior estimates) are provided in the Appendix. The estimation sample can be roughly split into two regimes. One is from 1959 to 1998, and the other spans the period 1999 to 2014 (see Figure B-3). To understand the key properties of the estimated dynamics, in the first and second panel of Figure 1, I report the response of consumption growth and inflation forecasts following a one-standard-deviation inflation shock.

Three aspects of the results are noteworthy. First, the sign of consumption's reaction to an inflation shock changed from negative to positive over the last 15 years: A one-standard-deviation inflation surprise predicts that consumption growth will be higher by approximately 60 basis points (bps) in the next year. Inflation carries good news about consumption growth. Second, the own-shock responses for inflation decayed much faster over the last 15 years: The impact of a one-standard-deviation inflation surprise on itself completely dies out over the next six months. This is mainly due to a large decline in the persistence of the expected inflation process; for example, the autoregressive coefficient for inflation dropped from 0.99 to 0.15 (refer to Appendix for details). Third, there is significant reduction in the variance of inflation innovations. Overall, the key aspects of the data are that the inflation dynamics have substantially changed over time and there are periods in which an inflation shock can be good news for consumption growth.[10]

The third panel of Figure 1, in fact, shows that yields with longer maturities are, on average, higher than those with shorter maturities. The perspective of existing term structure models is puzzling in that during periods from 1999 to 2014, in which inflation is procyclical, the Treasury yield curve (while shifted down significantly) still slopes upward.[11] That the correlation between bond and stock returns changed from positive to negative in those periods (the fourth panel of Figure 1) is a particularly interesting observation. The result is consistent with recent empirical studies that U.S. Treasury bonds have served as a hedge to stock market risks in the last decade.[12]

The new set of evidence is interesting not only because it shows the limitations of the existing approaches but also because it implies that the sources of risk behind the yield curve might have changed over time. There is an important reason to believe that the yield curve and inflation dynamics are sensitive to monetary policy shifts or changes in the distribution of economic shocks.[13] Despite the extensive study on bond markets, only few papers try to investigate the origins of the bond market changes. The suggested hypotheses fall into two broad categories. The first view attributes the cause of the bond market changes to shift in the correlation between the nominal and real disturbances, for example, Campbell, Sunderam, and Viceira (2016), David and Veronesi (2013), and Ermolov (2015). The second view argues that the root cause is changes in the conduct of monetary policy (see Campbell, Pflueger, and Viceira (2015)). This paper puts forward a unified framework that enables joint assessment of the strength of these two hypotheses which in fact are not mutually exclusive. In sum, the new stylized empirical facts posit the need to look at the data from a broader perspective, which calls for a more flexible approach to the joint modeling of macroeconomic fundamentals, monetary policy, and stock and bond asset prices. I turn to this in the next section.

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[10] This evidence is also documented by David and Veronesi (2013).

[11] As shown in Campbell (1986), positive correlation in consumption growth and inflation implies a downward-sloping nominal yield curve.

[12] See Baele, Bekaert, and Inghelbrecht (2010), Campbell, Pflueger, and Viceira (2015), Campbell, Sunderam, and Viceira (2016), and David and Veronesi (2013).

[13] See Ang et al. (2011), Bikbov and Chernov (2013), and Shaliastovich and Yamarthy (2015).

2. Model

I develop an asset pricing model that embeds risks through shifts in the strength with which the Federal Reserve tries to pursue its stabilization policy, as well as in the covariance matrix of nominal inflation target and real growth innovations. The real side of the model builds on the long-run risks model of Bansal and Yaron (2004) and assumes that consumption growth contains a small predictable component (i.e., long-run growth), which, in conjunction with investors' preference for an early resolution of uncertainty, determines the price of real assets. The nominal side of the model extends the model of Gallmeyer et al. (2007) in that inflation dynamics are endogenously derived from the monetary policy rule, and the nominal assets inherit the properties of monetary policy. As a consequence of my model features, cyclical properties of inflation and bond price dynamics depend on changes in monetary policy aggressiveness and the distributions of macroeconomic shocks.

2.1 Preferences

I consider an endowment economy with a representative agent that has recursive preferences and maximizes her lifetime utility,

$$V_t = \max_{C_t} \left[(1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(\mathbb{E}_t [V_{t+1}^{1-\gamma}] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}},$$

subject to the budget constraint

$$W_{t+1} = (W_t - C_t) R_{c,t+1},$$

where W_t is the wealth of the agent, $R_{c,t+1}$ is the return on all invested wealth, γ is risk aversion, $\theta = \frac{1-\gamma}{1-1/\psi}$, and ψ is intertemporal elasticity of substitution. The log of the real stochastic discount factor (SDF) is

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}. \tag{2}$$

2.2 Exogenous endowment process and inflation target rule

Following Bansal and Yaron (2004), I decompose consumption growth, Δc_{t+1}, into a (persistent) long-run growth component, $x_{c,t}$, and a (transitory) short-run component, $\sigma_c \eta_{c,t+1}$. The persistent long-run growth component is modeled as an AR(1) process. The inflation target, $x_{\pi,t}$, is modeled by a random walk process. This is an identifying assumption that permanent changes in realized inflation cannot occur without changes in the inflation target of the Federal Reserve.

The covariance between the inflation target shock and the real growth shock, which is captured by $\beta(S_t)\sigma_{xc}^2(S_t)$, is not zero and is subject to sign switches. Here, S_t denotes the regime indicator variable $S_t \in \{1, ..., N\}$. In essence, the regime-switching $\beta(S_t)$ in the covariance term attempts to capture possible structural shifts in the economy in reduced form. The economy in which the value of β is negative raises the relative importance of supply shocks and, importantly, translates adverse supply shocks into more persistent positive movements in the inflation target itself. On the other hand, the positive value of β works to increase the relative role of demand shocks and decreases (increase) the impact of adverse supply (favorable demand) shocks on the inflation target. I refer to Appendix E.18 for a thorough discussion about this interpretation and how one should think about inflation target shocks being correlated with endowment shocks.

Dividend streams, Δd_{t+1}, have levered exposures to $x_{c,t}$, for which magnitude is governed by the parameter ϕ. I allow $\sigma_d \eta_{d,t+1}$ to capture the idiosyncratic movements in dividend streams. Put together, the joint dynamics for the cash flows, $G_t = [\Delta c_t, \Delta d_t]'$, are

$$G_{t+1} = \mu + \varphi X_t + \Sigma \eta_{t+1}, \quad \eta_{t+1} \sim N(0, I),$$
$$X_{t+1} = \Phi(S_{t+1}) X_t + \Omega(S_{t+1}) \Sigma_x(S_{t+1}) \eta_{x,t+1}, \quad \eta_{x,t+1} \sim N(0, I), \tag{3}$$

where $\mu = [\mu_c, \mu_d]'$, $\eta_t = [\eta_{c,t}, \eta_{d,t}]'$, $X_t = [x_{c,t}, x_{\pi,t}, x_{i,t}]'$, $\eta_{x,t} = [\eta_{xc,t}, \eta_{x\pi,t}, \eta_{xi,t}]'$ and[14]

$$\varphi = \begin{bmatrix} 1 & 0 & 0 \\ \phi & 0 & 0 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_c & 0 \\ 0 & \sigma_d \end{bmatrix},$$

$$\Phi = \begin{bmatrix} \rho_c & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \rho_i \end{bmatrix}, \quad \Omega = \begin{bmatrix} 1 & 0 & 0 \\ \beta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Sigma_x = \begin{bmatrix} \sigma_{xc} & 0 & 0 \\ 0 & \sigma_{x\pi} & 0 \\ 0 & 0 & \sigma_{xi} \end{bmatrix}.$$

$x_{i,t}$ is monetary policy shock that follows an AR(1) process (explained later).

2.3 Exogenous monetary policy rule

Monetary policy consists of two components: stabilization and a time-varying inflation target. I assume that the Federal Reserve makes informed decisions about inflation fluctuations at different frequencies. While the Federal Reserve attempts to steer actual inflation toward the inflation target (which itself is time varying) at low frequencies, it aims to stabilize inflation fluctuations relative to its target at high frequencies. In particular, I assume that the strength with which the central bank tries to pursue its goal—a stabilization policy—changes over time along the lines explored by Clarida, Gali, and Gertler (2000). Stabilization policy is "active" ($\tau_\pi > 1$) or "passive" ($\tau_\pi \leq 1$), depending on its responsiveness to inflation fluctuations relative to the target.

[14] The variance-covariance matrix $\Omega(S_t)\Sigma_x(S_t)\Sigma_x(S_t)'\Omega(S_t)'$ is in this particular form because the variance of the real growth shocks is independent of $\beta(S_t)$.

In sum, monetary policy follows a regime-switching Taylor rule,

$$i_t = \tau_0(S_t) + \underbrace{\tau_c(S_t)x_{c,t}}_{\text{real growth}} + \underbrace{\tau_\pi(S_t)(\pi_t - \Gamma_0 - x_{\pi,t})}_{\text{inflation around target}} + \underbrace{x_{\pi,t}}_{\text{target}} + \underbrace{x_{i,t}}_{\text{policy shock}}, \tag{4}$$

where $\tau_c(S_t)$ and $\tau_\pi(S_t)$ capture the central bank's reaction to real growth and to the variation in short-run inflation, respectively.

Several important features should be discussed. (4) assumes that monetary policymakers pursue a policy with a time-varying inflation target. In accord with much of the previous literature, I argue that large swings in inflation in the 1970s and the trending down of the 1980s could not have occurred without ongoing shifts in the Federal Reserve's inflation target.[15] In the context of the term structure models, it is important to consider an explicit role for the inflation target since the target behaves similar to a level factor of the nominal term structure. The specification of (4) resembles specifications in which the level factor of the term structure directly enters into the monetary policy rule (see Rudebusch and Wu (2008), for example). Second, while policy rule inertia is a more plausible description of U.S. monetary policy actions (see discussions in Coibon and Gorodnichenko (2011)), it is assumed to be absent. (4) allows me to apply Davig and Leeper (2007)'s solution method and characterize inflation as exact affine functions of the "current" state variables, X_t, without any "lagged" term.[16] Having said that, I acknowledge that caution has to be taken since the specification of the monetary policy rule can be viewed as too simplistic. I provide several robustness checks in Appendix E.17 and argue that the empirical results do not seem to be driven by the absence of policy rule inertia.

2.4 Markov chain

To achieve flexibility while maintaining parsimony, I assume that the model parameters evolve according to a three-state Markov chain, $S_t \in \{1,2,3\}$:

1. Countercyclical Macroeconomic Shocks and Active Monetary Policy (CA): $\beta < 0$, $\tau_\pi > 1$,
2. Countercyclical Macroeconomic Shocks and Passive Monetary Policy (CP): $\beta < 0$, $\tau_\pi \leq 1$,
3. Procyclical Macroeconomic Shocks and Active Monetary Policy (PA): $\beta \geq 0$, $\tau_\pi > 1$.

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[15] Note that incorporating a time-varying inflation target is quite common in the monetary policy literature (see Coibon and Gorodnichenko (2011); Aruoba and Schorfheide (2011); and Davig and Doh (2014)). The exogenous inflation target process is a shortcut. For example, the learning models of Sargent (1999) or Primiceri (2006) imply that inflation target varies over time because the Federal Reserve learns about the output-inflation trade-off and tries to set inflation optimally. This paper does not explore such mechanism and assumes for simplicity that inflation target varies exogenously, which is common in much of the monetary policy literature (see Ireland (2007); Campbell, Pflueger, and Viceira (2015); and Del Negro, Giannoni, and Schorfheide (2015)).

[16] Rudebusch (2002) argues that, to study the term structure, it seems sensible to consider the monetary policy rule without an interest-rate-smoothing motive. Based on the term structure evidence, Rudebusch (2002) shows that monetary policy inertia is not due to the smoothing motive but is due to persistent shocks.

I define a Markov transition probability matrix by Π, which summarizes all 3^2 transition probabilities. The labeling of the regimes is explained in detail for the asset pricing implications.

2.5 Endogenous inflation process and determinacy of equilibrium

Inflation dynamics can be endogenously determined from the monetary policy rule (4) and an asset-pricing equation, which is given below,

$$i_t = -E_t[m_{t+1} - \pi_{t+1}] - \frac{1}{2} Var_t[m_{t+1} - \pi_{t+1}]. \tag{5}$$

Substituting the asset-pricing equation (5) into the monetary policy rule (4), the system reduces to a single regime-dependent equation

$$\tau_\pi(S_t)\pi_t = E_t[\pi_{t+1}] + \Lambda_0(S_t) + \Lambda_1(S_t)X_t, \tag{6}$$

where $\Lambda_0(S_t)$ and $\Lambda_1(S_t)$ are function of the model structural parameters.

I posit regime-dependent linear solutions of the form

$$\pi_t = \Gamma_0(S_t) + \Gamma_1(S_t)X_t. \tag{7}$$

For ease of exposition, I introduce a diagonal matrix Ψ, where the ith diagonal component is $\tau_\pi(i)$. According to Proposition 2 of Davig and Leeper (2007), a unique bounded solution (determinate equilibrium) exists provided that the "long-run Taylor principle" (summarized by the two conditions) is satisfied:

1. $\tau_\pi(i) > 0$, for $i \in \{1,2,3\}$,
2. All the eigenvalues of $\Psi^{-1} \times \Pi$ lie inside the unit circle.

A detailed derivation is provided in Appendix C.8.

In a fixed-regime environment, the equilibrium inflation is not unique, and multiple solutions exist, including stationary sunspot equilibria when monetary policy is passive, $\tau_\pi \leq 1$. A striking feature is that with regime switching, there exists determinate equilibrium, even with passive monetary policy. Figure 2 provides admissible ranges (black-shaded regions) of monetary policy coefficients consistent with the long-run Taylor principle. According to Figure 2, monetary policy can be passive some of the time, as long as the passive regime is sufficiently short-lived (see discussion in Baele et al. (2015); Davig and Leeper (2007); Foerster (2016)). Allowing for (short-lived) passive monetary policy has several important asset pricing implications that I discuss shortly.

2.6 Neutrality of monetary policy

I rearrange real consumption growth process in (3) and inflation process in (7) to discuss the neutrality of monetary policy,

$$\begin{bmatrix} \Delta c_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_c \\ \Gamma_0(S_{t+1}) \end{bmatrix} + \begin{bmatrix} e_1 \\ \Gamma_1(S_{t+1})\Phi(S_{t+1}) \end{bmatrix} X_t + \begin{bmatrix} \sigma_c \eta_{c,t+1} \\ \Gamma_1(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1} \end{bmatrix} \tag{8}$$

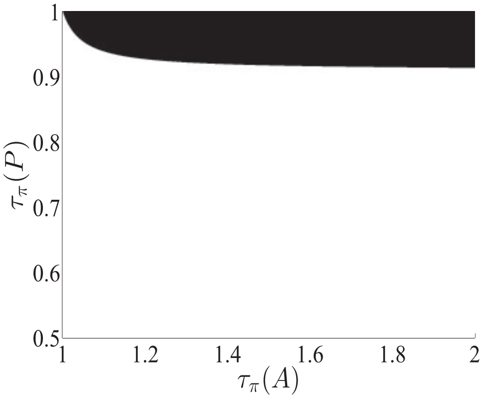

Figure 2
Determinacy regions
Parameter combinations in the black-shaded regions imply a unique equilibrium in the regime-switching model. I fix $\Pi = \hat{\Pi}$ at their posterior median estimates (15) and vary $0.5 \leq \tau_\pi(P) \leq 1$ and $1 \leq \tau_\pi(A) \leq 2$ to compute the determinacy regions.

where $e_1 = [1,0,0]$. In this environment, monetary policy is assumed to be "neutral"; that is, the federal funds rate neither stimulates nor restrains real endowment growth (note that $e_1 X_t$ is independent from monetary policy).[17] This simplifying assumption enables a sophisticated characterization of the formation of inflation expectations.

2.7 Expectations formation

The implicit assumption in most studies of monetary policy behavior (and its transmission mechanism to asset prices) is that monetary policy shifts, if any, are unanticipated, and agents naively believe the new regime is permanent (for example, see Campbell, Pflueger, and Viceira (2015)). Inflation in (8) has a purely forward-looking specification that reflects agents' beliefs that regime change is possible. There is ample evidence in the literature that suggests that the U.S. economy was subject to occasional regime switches (empirical evidence will soon follow). This paper is a step toward bringing theory in line with evidence and has a potential of evaluating the consequences of assuming absorbing monetary policy regime.

2.8 Notations

Before I explain the solution of the model, I introduce some notations. $r_{c,t+1}$ is the log real return of the consumption claim. $r_{m,t+1}$ denotes the log real stock market return. I use \$ to distinguish nominal from real values. The nominal n-maturity log zero-coupon bond price is $p^{\$}_{n,t}$, and the respective log bond

[17] The proposed framework resembles a classical monetary model with fully flexible prices. Readers are referred to Chapter 2 of Gali's book.

yield is $y^{\$}_{n,t} = -\frac{1}{n} p^{\$}_{n,t}$. $r^{\$}_{n,t+1}$ denotes the log return to holding a n-maturity nominal bond from t to $t+1$. $rx^{\$}_{n,t+1}$ is the log return to holding a n-maturity nominal bond from t to $t+1$ in excess of the log return to a one period nominal bond. $\xi^{\$}_{n,t}$ is the term premium for the nominal n-maturity bond, which is the risk compensation for holding longer maturity bonds relative to short maturity bonds.

2.9 Asset solutions and asset pricing implication

The first-order condition of the agent's expected utility maximization problem yields the Euler equations

$$E_t\left[\exp(m_{t+1} + r_{k,t+1})\right] = 1, \quad k \in \{c, m\}, \quad \text{Real Assets}, \tag{9}$$

$$p^{\$}_{n,t} = \log E_t[\exp(m_{t+1} - \pi_{t+1} + p^{\$}_{n-1,t+1})], \quad \text{Nominal Assets}. \tag{10}$$

The solutions to (9) and (10) depend on the joint dynamics of consumption and dividend growth (3) and inflation (7). Asset prices are determined from the approximate analytical solution described by Bansal and Zhou (2002), who assume that asset prices are affine function of state X_t conditional on regime S_t. The appendix provides the details.

For the sake of exposition, I set monetary policy shock to zero and reduce the state variables from three to two: real growth and inflation target.[18] The nominal n-maturity log bond price is an affine function of the state conditional on the current regime S_t (here I omit S_t for simplicity)

$$p^{\$}_{n,t} = C^{\$}_{n,0} + C^{\$}_{n,1} X_t, \tag{11}$$

where $C^{\$}_{n,1} = [C^{\$}_{n,1,c}, C^{\$}_{n,1,\pi}]$ and $X_t = [x_{c,t}, x_{\pi,t}]'$. The respective nominal bond yield loadings can be expressed by

$$B^{\$}_{n,1,c} = -\frac{1}{n} C^{\$}_{n,1,c} = \left(\frac{1/\psi\,\tau_\pi - \rho_c \tau_c}{\tau_\pi - \rho_c}\right) \frac{1}{n} \left(\frac{1-\rho_c^n}{1-\rho_c}\right),$$

$$B^{\$}_{n,1,\pi} = -\frac{1}{n} C^{\$}_{n,1,\pi} = 1. \tag{12}$$

These are the solution coefficients in the absence of regime switching. Note that $B^{\$}_{n,1,c}$ decays to zero for long maturity bonds, and $B^{\$}_{n,1,\pi}$ is always one, implying that any change in inflation target induces parallel shifts in the entire yield curve. Under $\frac{1}{\psi}\min\{1, \tau_\pi/\rho_c\} > \tau_c$, the sign of bond yield loading $B^{\$}_{n,1,c}$ is positive if $\tau_\pi > 1$, that is, monetary policy is active and negative when monetary policy is passive, $\tau_\pi \leq 1$.[19] When monetary policy is active (passive), bond prices rise

18 Since monetary policy shock is orthogonal to the real growth and inflation target shocks, its role in the asset markets is not as important as that of the previous two shocks. I am shutting down monetary policy shock for the purpose of providing intuition of the model.

19 The sign of $B^{\$}_{n,1,c}$ depends on the relative magnitude of τ_π and ρ_c, and I assume that ρ_c is fairly close to one in this analysis.

(fall) in response to decrease in real growth and bond yields become procyclical (countercyclical).

After some tedious algebra, the sign of the one-period expected excess return of a n-maturity nominal bond (bond risk premium) is expressed as

$$\text{sign}\Big(E_t(rx^{\$}_{n,t+1})+\frac{1}{2}Var_t(rx^{\$}_{n,t+1})\Big)\approx -\text{sign}\left((B^{\$}_{n-1,1,c}+\beta)\frac{(\gamma-1/\psi)\kappa_1}{1-\kappa_1\rho_c}\right). \tag{13}$$

The approximation is accurate for highly persistent real growth process, ρ_c, and the Campbell-Shiller log approximation constant, κ_1. Similarly, the sign of the conditional correlation between the real stock market and the n-maturity nominal bond return is characterized by

$$\text{sign}\Big(Corr_t\big(r_{m,t+1},rx^{\$}_{n,t+1}\big)\Big)=-\text{sign}\big(B^{\$}_{n-1,1,c}+\beta\big). \tag{14}$$

I refer to Appendices C.9 and C.10 for the exact expression.

To build intuition into (13) and (14), I start by considering the limiting case of a fixed-regime economy. Throughout the analysis, I assume that agents have a preference for an early resolution of uncertainty ($\gamma > 1/\psi$). To facilitate intuition, I start with $\beta=0$. Suppose if monetary policy is active, then nominal bonds are hedges ($B^{\$}_{n-1,1,c}\geq 0$) and bond risk premium falls in response to increase in real growth and inflation target uncertainty. In this environment, nominal bonds are qualitatively similar to real bonds.[20] The implied stock-bond return correlation is negative. The same could be said for the reverse logic: The signs of bond risk premium and stock-bond return correlation flip and become positive under passive monetary policy regime.

The introduction of $\beta\neq 0$ complicates the analysis. Suppose that monetary policy is active. As long as the covariance of inflation target and real growth shocks is small in magnitude, $\beta\geq -B^{\$}_{n-1,1,c}$, the implication on bond risk premium and stock-bond return correlation will be identical as before. In such case, the economy experiences "procyclical" macroeconomic shocks. For the sake of labeling purpose, the respective regime is PA. However, a sufficiently large "countercyclical" macroeconomic shocks, captured by $\beta < -B^{\$}_{n-1,1,c}$, can reverse the sign and generate positive bond risk premium and stock-bond return correlation. Here, the regime is CA. Suppose now that monetary policy is passive. Following large countercyclical macroeconomic shocks (which are bad news for the economy), the implied bond risk premium and stock-bond return correlation are positive and largest in magnitude across all cases. This environment, called the CP regime, is the extreme version of the economy described by Piazzesi and Schneider (2006); Wachter (2006); Eraker (2008); Bansal and Shaliastovich (2013), who assume inflation is countercyclical and risky.

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[20] When agents have a preference for an early resolution of uncertainty ($\gamma > 1/\psi$), real bonds are hedges against low growth and real bond risk premiums are always negative. Because these hedging effects are stronger at longer horizons, this implies a downward-sloping real term structure. The equation for real bond risk premium is provided in (E21), and Table E-5 displays the model-implied real term structure.

Table 1
Asset pricing implications in a fixed-regime economy

		CA	CP	PA
Unique inflation solution		Yes	No	Yes
Bond loading	$B^{\$}_{n,1,c}$	+	−	+
Stock-bond return correlation	$Corr_t\left(r_{m,t+1}, r^{\$}_{n,t+1}\right)$	+	+	−
Bond risk premium	$E_t(rx^{\$}_{n,t+1}) + \frac{1}{2}Var_t(rx^{\$}_{n,t+1})$	+	+	−

Notes: The regimes are labeled as (1) the CA regime, countercyclical macroeconomic shocks, and active monetary policy; (2) the CP regime, countercyclical macroeconomic shocks, and passive monetary policy; and (3) the PA regime, procyclical macroeconomic shocks, and active monetary policy.

Table 1 summarizes the model's intuition. In general, the signs of relevant asset pricing moments are unambiguously determined in the CP and PA regimes, while they are not in the CA regime. They ultimately depend on the distribution of macroeconomic shocks (captured by β) and the aggressiveness of the monetary policy (captured by τ_π). The signs will be determined only for sufficiently large countercyclical macroeconomic shocks, that is, $\beta < -B^{\$}_{n-1,1,c}$, which is assumed in Table 1.

Until now, I have characterized each of the "within-regime" risks. However, it is sensible to argue that the economic agents anticipate future regime changes and the associated regime risks are reflected in today's asset market. This is called "across-regime" risks, or the extent to which the risk properties of alternative regimes are incorporated due to regime-switching dynamics. With regime switching, it is difficult to understand the asset pricing implications because all regime risks are mixed together through the iterated expectation over the regimes. Determining what the driving forces behind the changes in the asset markets is entirely an empirical question. To answer this, I now turn to the estimation part of the model.

3. Empirical Results

The data set used in the empirical analysis is described in Section 3.1. Bayesian inference is discussed in Section 3.2. Section 3.3 discusses parameter restrictions of the model and identification of the regime. Section 3.4 discusses parameter estimates and regime probabilities. The model's implications for macroeconomic aggregates and asset prices are explained in Section 3.5. Finally, Section 3.6 discusses model caveats and provides various robustness checks.

3.1 Data

Monthly consumption data represent per capita series of real consumption expenditures on nondurables and services from the National Income and Product Accounts (NIPA) tables, which are available from the Bureau of Economic Analysis. Aggregate stock market data consist of monthly observations of returns, dividends, and prices of the CRSP value-weighted portfolio of all stocks traded on the NYSE, AMEX, and NASDAQ. Price and dividend series are constructed on a per share basis, like in Campbell and Shiller (1988b); Hodrick (1992). Market data are converted to real data using the consumer price index (CPI) from the Bureau of Labor Statistics. Growth rates of consumption and dividends are constructed by taking the first difference of the corresponding log series. Inflation represents the log difference of the CPI. Monthly observations of U.S. Treasury bonds with maturities at one to five years are from CRSP. The time series spans the monthly data from 1963:M1 to 2014:M12. The appendix provides a detailed description of the data.

3.2 Bayesian inference

Posterior inference is implemented with a Metropolis-within-Gibbs sampler (see the previous work of Carter and Kohn (1994); Kim and Nelson (1999)). $Y_{1:T}$ denotes the sequence of observations, where

$$Y_t = (\Delta c_t, \pi_t, pd_t, y_{1,t}, y_{2,t}, y_{3,t}, y_{4,t}, y_{5,t}).$$

Moreover, let $S_{1:T}$ be the sequence of hidden states, and let $\Theta = (\Theta_1, \Theta_2)$, where

$$\begin{aligned}
&\Theta_1 = (\delta, \gamma, \psi),\\
&\Theta_2 = (\mu_c, \mu_d, \rho_c, \rho_i, \phi, \sigma_c, \sigma_d, \sigma_{xc}, \sigma_{x\pi}, \sigma_{xi}, \beta(-), \beta(+),\\
&\qquad \tau_0(P), \tau_0(A), \tau_c(P), \tau_c(A), \tau_\pi(P), \tau_\pi(A)),\\
&\Theta_\Pi = (\{\Pi_{ij}\}_{i,j=\{1,2,3\}}).
\end{aligned}$$

Metropolis-within-Gibbs algorithm involves iteratively sampling from three conditional posterior distributions. Details are provided in Appendix C.14.

3.3 Identification and parameter restriction

As discussed before, the identification of the regime is achieved by segmenting the economy into the following three cases:

1. Countercyclical Macroeconomic Shocks and Active Monetary Policy (CA): $\beta < 0$, $\tau_\pi > 1$,
2. Countercyclical Macroeconomic Shocks and Passive Monetary Policy (CP): $\beta < 0$, $\tau_\pi \le 1$,
3. Procyclical Macroeconomic Shocks and Active Monetary Policy (PA): $\beta \ge 0$, $\tau_\pi > 1$.

I allow the standard deviation of the inflation target innovations $\sigma_{x\pi}$ to differ across regimes. In particular, I assume that while $\sigma_{x\pi}$ is largest under passive monetary policy regime, it is assumed to be smallest under procyclical inflation regime.[21] The restriction is summarized by

$$\sigma_{x\pi}(CP) > \sigma_{x\pi}(CA) > \sigma_{x\pi}(PA).$$

[21] The restriction on $\sigma_{x\pi}$ helps identify the different regimes. However, I find that even without this restriction the identified regimes are qualitatively the same.

Table 2
Posterior median estimates

(1) Preference		(2) Consumption		(3) Dividend		(4) Factor shocks		(5) Monetary policy	
δ	0.999	μ_c	0.0016	μ_d	0.0016	$\beta(-)$	−2.50 [−3.81, −1.32]	$\tau_0(A)$	0.0041 [0.0021, 0.0047]
ψ	2	ρ_c	.99	ϕ	3	$\beta(+)$	1.00 [0.08, 2.13]	$\tau_0(P)$	0.0043 [0.0031, 0.0052]
γ	8	σ_c	0.0024	σ_d/σ_c	6.25	σ_{xc}	0.00015 [0.00011, 0.00018]	$\tau_\pi(A)$	1.40 [1.11, 1.75]
						$\sigma_{x\pi}(CA)$	0.00019 [0.00017, 0.00023]	$\tau_\pi(P)$	0.94 [0.89, 0.99]
						$\sigma_{x\pi}(CP)$	0.00039 [0.00034, 0.00047]	$\tau_c(A,P)$	0.48 [0.38, 0.57]
						$\sigma_{x\pi}(PA)$	0.00011 [0.00008, 0.00014]		
						σ_{xi}	0.0020 [0.0013, 0.0027]		

Notes: The estimation results are based on monthly data from 1963:M1 to 2014:M12. A subset of parameters under (1), (2), and (3) is fixed based on Schorfheide, Song, and Yaron (2016). I show the posterior interquartile range (5%, 95%) in brackets.

Since real endowment process is exogenously specified in this economy, I assume that policy response to real growth is identical across regimes, that is, $\tau_c(A)=\tau_c(P)$.

Finally, to reduce the number of estimated parameters, a subset of parameters, under (1), (2), and (3) in Table 2, is fixed based on Schorfheide, Song, and Yaron (2016). I also assume that the monetary policy shock is not serially correlated, that is, $\rho_i=0$. This restriction is conservative in terms of the fit of the model because it effectively reduces the number of persistent state variables from three to two: real growth and inflation target.

3.4 Parameter estimates and regime probabilities

The priors for the parameters are fairly agnostic and are shown in Appendix C.14. Percentiles for the posterior distribution are reported in Table 2. The most important results for the subsequent analysis are provided in (15) and in the fourth and fifth columns of Table 2.

$$\Pi=\begin{bmatrix} \underset{[0.85,0.97]}{0.907} & \underset{[0.035,0.067]}{0.045} & \underset{[0.042,0.071]}{0.050} \\ \underset{[0.042,0.071]}{0.050} & \underset{[0.85,0.97]}{0.911} & \underset{[0.042,0.070]}{0.045} \\ \underset{[0.037,0.068]}{0.049} & \underset{[0.035,0.067]}{0.046} & \underset{[0.85,0.97]}{0.905} \end{bmatrix}. \tag{15}$$

First, (15) reports posterior estimates of the Markov-chain transition probabilities. Below each posterior median parameter estimate, I show the posterior interquartile range (5%, 95%) in brackets.[22] The regimes are ordered by CA, CP, and PA. The respective unconditional regime probabilities are

[22] Note that posterior median values do not necessarily sum to one.

0.35, 0.33, and 0.32. This result can be interpreted as an indication that the risks of experiencing countercyclical macroeconomic shocks are substantial, as indicated by the sum of the probability of the CA and CP regimes, 0.68. The unconditional probability of staying in the active monetary policy regime, as indicated by the sum of the probability of the CA and PA regimes, is around 0.67, which is twice as large as that of the passive monetary policy regime.

Second, strong evidence suggests parameter instability in the dynamics of the long-run components. Most prominently, the sign change in the correlation structure is notable: the posterior median estimate of β is -2.5 in the countercyclical macroeconomic shock regime and 1.0 in the procyclical macroeconomic shock regime; and the correlation between real growth and inflation target $\beta\sigma_{xc}/\sqrt{\beta^2\sigma_{xc}^2+\sigma_{x\pi}^2}$ is -0.9, -0.7, and 0.8 under the CA, CP, and PA regimes, respectively.

Third, two very different posterior estimates of the monetary policy rule in the fifth column of Table 2 support the view of Clarida, Gali, and Gertler (2000) that there has been a substantial change in the way monetary policy is conducted. Active regime is associated with a larger monetary policy rule coefficient, 1.40, which implies that the central bank will more aggressively respond to short-run inflation fluctuations. Passive regime is characterized by a less responsive monetary policy rule, in which I find much lower loading, 0.94. Given the posterior transition probabilities, I verify that the estimated monetary policy coefficients fall into the admissible ranges consistent with the long-run Taylor principle in Figure 2.

Figure 3 depicts the smoothed posterior regime probabilities. The estimation identifies inflation as countercyclical from the early 1970s through the late 1990s and as procyclical from the late 1990s onward. This is consistent with the evidence provided in Figure 1. Figure 3 also suggests that the switch is not a permanent event, but is an occasional one.[23] The historical paths of the monetary policy stance are also consistent with the empirical monetary literature: Active monetary policy appeared in the early 1960s but was largely dormant until the early 1980s; it became active after the appointment of Paul Volcker as Chairman of the Federal Reserve and remained active throughout the sample.[24]

3.5 Implications for macroeconomic aggregates and asset prices

While asset pricing moments implicitly enter the likelihood function of the state-space model, it is instructive to examine the extent to which sample moments implied by the estimated state-space model mimic the sample moments computed from the actual data set. To do so, I report percentiles

[23] This evidence is also supported by David and Veronesi (2013).

[24] The smoothed paths for monetary policy are broadly consistent with those found in Baele et al. (2015); Bianchi (2012); Clarida, Gali, and Gertler (2000); Ang et al. (2011); Bikbov and Chernov (2013); Coibon and Gorodnichenko (2011).

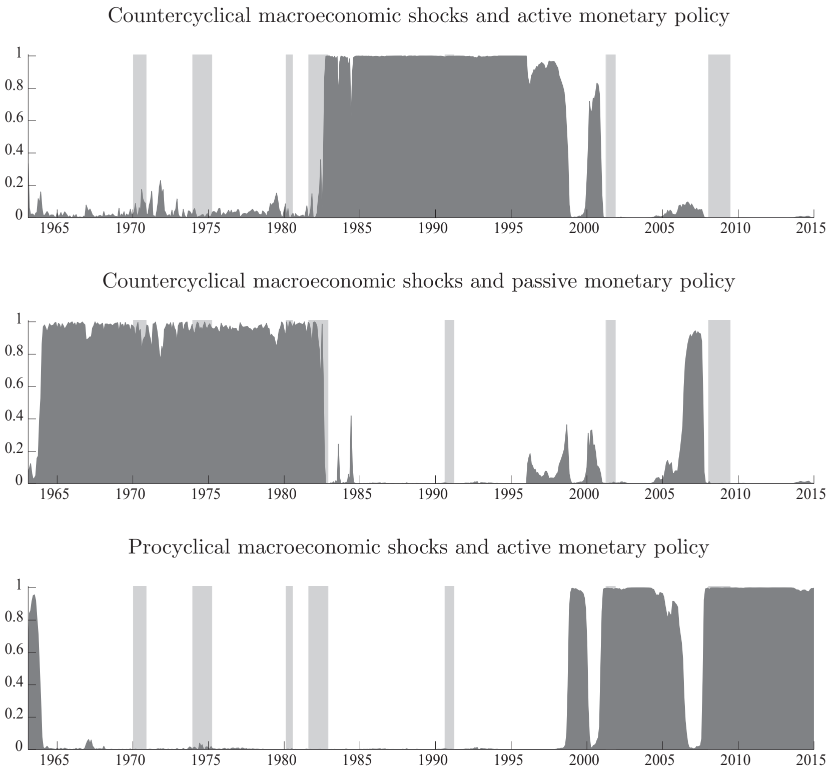

Figure 3
Regime probabilities: Macro and asset data
The dark-gray-shaded areas represent posterior medians of regime probabilities. The light-gray-shaded bars indicate the NBER recession dates.

of the posterior predictive distribution for various sample moments based on simulations from the posterior distribution of the same length as the data.[25] The posterior predictive distributions are obtained conditional on posterior median estimates of the parameters and only reflect sampling uncertainty. Except for a few cases, to facilitate comparison across different regimes, I only report the median values in the tables. Broadly speaking, I compare two cases: one in which regimes are fixed and the other in which regime switching is allowed in the economy. When regime switching is allowed, the solutions of the model account for "across-regime" risks through the iterated expectation over the regimes. When simulation allows for actual transition to different regimes, the results are reported under "Mix." If simulation results under regime switching are reported under a specific regime identity, for example, "CA," this means that, even though agents take into account possible regime switches in their

25 This is called a posterior predictive check; see Geweke (2005) for a textbook treatment.

Table 3
Model-implied moments: Macroeconomic aggregates

	Data	Regime-switching model				Fixed-regime model		
		Mix	CA	CP	PA	CA	CP	PA
Consumption								
$E(\Delta c)$	1.90	1.93	1.93	1.86	1.87	1.94	1.90	1.90
$\sigma(\Delta c)$	0.93	0.85	0.86	0.85	0.85	0.88	0.88	0.88
$AC(\Delta c)$	0.06	0.10	0.11	0.09	0.09	0.11	0.09	0.10
$corr(\Delta c,\pi)$	−0.05	−0.04	−0.18	−0.09	0.14	−0.18	−0.14	0.14
Inflation								
$E(\pi)$	3.71	3.58	3.54	4.24	3.26	3.26	3.93	3.29
$\sigma(\pi)$	1.10	1.30	1.41	1.63	0.74	1.42	1.81	0.76
$AC(\pi)$	0.62	0.84	0.86	0.90	0.56	0.87	0.93	0.51
Expected inflation								
1-year ahead	3.60	3.32	3.42	3.30	3.26	3.21	3.44	3.27
5-year ahead	—	3.33	3.33	3.33	3.33	3.21	3.44	3.27

Notes: The one-year-ahead inflation forecasts for the Survey of Professional Forecasters are provided by the Federal Reserve Bank of Philadelphia and are available from 1970 to 2014. I report the mean of the forecasts. The appendix provides detailed descriptions of the data.

expectations formation, regime switches do not occur along the simulated paths ex post. By comparing these outcomes to those from the fixed-regime economy, I am able to isolate the expectations effect on asset prices.

3.5.1 Macroeconomic aggregates. The model-implied distributions for the first and second moments of the macroeconomic aggregates are provided in Table 3. Note that the second moment measures the amount of variation in sample data and does not reflect the unconditional variance.[26] The data moments for consumption are based on the measurement-error-free monthly consumption growth series from Schorfheide, Song, and Yaron (2016).[27] Because the likelihood function utilizes a broader set of moments which include the entire sequence of autocovariances, the model tracks mean, variance, as well as the empirical autocorrelation function well. The sample moments for consumption do not seem to differ across regimes. Yet the sample moments for inflation are quite different across regimes: inflation in the CP regime is most volatile and persistent, while inflation in the PA regime is the exact opposite; the sample correlation between consumption and inflation is positive only in the PA regime; and the averages of inflation are also lowest in the PA regime, which is about 1% lower than those in the CP regime. Based on the regime probabilities in Figure 3, I find that the model is able to account for significant changes in the inflation dynamics observed in the data (see Figure 1). As the economy shifted from the CP regime to the CA regime and to the PA regime, the variance and persistence of inflation decreased substantially. Overall, the model finds that

[26] With nonstationary components, the population variance does not exist.

[27] Table A-1 provides the moments for the actual monthly consumption growth series.

Table 4
Model-implied moments: Bond market

	Data	Model						
		Regime-switching				Fixed-regime		
Maturity		Mix	CA	CP	PA	CA	CP	PA
Average bond yield, $y^{\$}_{n,t}$								
1-year bond	5.26	5.28	5.18	5.33	4.92	4.89	5.05	4.47
2-year bond	5.47	5.43	5.27	5.40	4.92	4.99	5.34	4.28
3-year bond	5.65	5.58	5.30	5.47	4.97	5.24	5.59	4.11
4-year bond	5.81	5.76	5.38	5.52	5.01	5.46	5.79	3.94
5-year bond	5.92	5.84	5.41	5.63	5.12	5.69	6.01	3.75
Average one-period bond risk premiums, $E_t rx^{\$}_{n,t+1}$								
1-year bond	—	0.08	0.08	0.07	−0.07	0.10	0.13	−0.08
2-year bond	—	0.20	0.18	0.18	−0.16	0.24	0.29	−0.18
3-year bond	—	0.34	0.28	0.30	−0.25	0.38	0.45	−0.27
4-year bond	—	0.49	0.39	0.43	−0.33	0.52	0.61	−0.36
5-year bond	—	0.65	0.51	0.55	−0.41	0.66	0.77	−0.45
Average term premiums, $\xi^{\$}_{n,t}$								
1-year bond	0.16	0.17	0.09	0.17	−0.10	0.20	0.24	−0.15
2-year bond	0.39	0.22	0.16	0.27	−0.11	0.42	0.50	−0.34
3-year bond	0.58	0.30	0.22	0.37	−0.09	0.63	0.74	−0.52
4-year bond	0.75	0.37	0.27	0.42	−0.01	0.85	0.95	−0.69
5-year bond	0.90	0.42	0.34	0.55	0.04	1.05	1.16	−0.86
Average multi-period bond risk premiums, $\frac{1}{n}\sum_{j=1}^{n-2} E_t rx^{\$}_{n-j,t+1+j}$								
1-year bond	—	0.16	0.08	0.16	−0.09	0.19	0.23	−0.14
2-year bond	—	0.21	0.15	0.26	−0.10	0.41	0.49	−0.33
3-year bond	—	0.29	0.21	0.36	−0.08	0.62	0.73	−0.51
4-year bond	—	0.36	0.26	0.41	0.00	0.84	0.94	−0.68
5-year bond	—	0.41	0.33	0.54	0.05	1.04	1.15	−0.85

Notes: The appendix provides detailed descriptions of the data. Treasury term premiums estimates for maturities from one to five years from 1963 to the present are based on Adrian, Crump, and Moench (2013) and are available at the Federal Reserve Bank of New York.

inflation has become procyclical and less risky as the economy shifted towards active monetary policy and experienced procyclical macroeconomic shocks.

Importantly, the model shows that monetary policy does impact the expected inflation process (since shocks themselves do not impact the expectations). The one-year-ahead expected inflation in the CP regime is largest in a fixed-regime economy, but not in a regime-switching economy. Since real growth is a mean-reverting process and inflation target expectation is based on the random-walk forecast (hence is identical across regimes), the long-run expectation of inflation converges in a regime-switching economy. In sum, the first and second moments for consumption and inflation implied by the model replicate the actual counterparts well.

3.5.2 Bond yields, term premiums, and bond risk premiums. Now, to evaluate whether the model can reproduce key bond market features in the data, the model-implied distributions of bond yields, term premiums, and bond risk premiums are reported in Table 4. To recap, I define the term premium $\xi^{\$}_{n,t}$ for the n-maturity nominal bond by

$$y^{\$}_{n,t} = \frac{1}{n} E_t \sum_{i=0}^{n-1} y^{\$}_{1,t+i} + \xi^{\$}_{n,t} \tag{16}$$

and the one-period bond risk premium or expected excess return by $E_t rx^{\$}_{n,t+1}$.

The model performs well along this dimension since the model-implied median values are fairly close to their data estimates. Yet important distinctions arise across regimes. Let's first focus on the fixed-regime case. Under the estimated parameter configuration, I find that the term structure is upward- (downward-) sloping and term premiums and bond risk premiums are positive (negative) in the CA regime and the CP (PA) regime consistent with the implications in Table 1.

Once regime switching is allowed, the striking feature is that the model is able to generate an upward-sloping term structure while maintaining negative bond risk premiums in the PA regime. In this regime, term premiums switch signs from negative to positive for longer-term bonds. To understand this better, let us look at the relationship between the term premium and the risk premium. In Appendix C.13, I show that the n-maturity nominal bond yield can be expressed as the sum of expected future bond returns over the entire holding period. Since the expected return can be written as the sum of the (1-maturity) nominal rate and excess return, the following identity holds

$$y^{\$}_{n,t} = \frac{1}{n} E_t \sum_{i=0}^{n-1} y^{\$}_{1,t+i} + \frac{1}{n} \sum_{j=0}^{n-2} E_t rx^{\$}_{n-j,t+1+j}. \tag{17}$$

From (16) and (17) it is straightforward to see that the term premium is the average of expected future bond risk premiums of declining maturity[28]

$$\xi^{\$}_{n,t} = \frac{1}{n} E_t rx^{\$}_{n,t+1} + \frac{1}{n} \sum_{j=1}^{n-2} E_t rx^{\$}_{n-j,t+1+j}. \tag{18}$$

The last panel of Table 4 provides the model-implied average multi-period bond risk premiums, that is, the second component of (18). In the regime-switching model the component turns positive, and, consequently, leads to positive term premium for longer-term bonds. When we look at the long end of the yield curve, we are effectively averaging over different regimes since all of them are likely to occur during the next years. This includes the CP regime which is characterized with both higher inflation level and uncertainty. The reverse holds true as well. In the CA or CP regime, the magnitudes of term premiums and one-period

[28] I thank an anonymous referee for pointing this out and providing insightful suggestions.

Table 5
Model-implied moments: Bond excess return predictability

Campbell-Shiller regression: Slope

	Data	Regime-switching model		
Maturity		5%	50%	95%
2-year bond	−0.62	−0.63	−0.49	−0.37
3-year bond	−1.01	−1.22	−1.00	−0.68
4-year bond	−1.42	−1.89	−1.50	−0.96
5-year bond	−1.45	−2.44	−1.96	−1.34
Cochrane-Piazzesi regression: R^2				
2-year bond	0.19	0.32	0.42	0.49
3-year bond	0.20	0.20	0.27	0.33
4-year bond	0.23	0.15	0.22	0.30
5-year bond	0.21	0.13	0.19	0.28

Notes: The model-implied distributions for the slope coefficient from the term spread regression of Campbell and Shiller (1991) are reported in the top panel. The model-implied R^2 values from the excess bond return regression of Cochrane and Piazzesi (2005) are reported in the second panel.

bond risk premiums are much smaller than those implied from the fixed-regime model. This is because the PA regime risk is mixed together through the iterated expectation over the regimes. In addition to the term premium channel, the average of future expected 1-maturity nominal rates, $\frac{1}{n}E_t\sum_{i=0}^{n-1} y^{\$}_{1,t+i}$, mean-reverts back up to a higher level in the PA regime.[29] Put together, increase in both the term premium and the average expected short rate components contribute toward an upward-sloping term structure. The key takeaway is that the expectations formation effect can go a long way in modifying equilibrium outcomes and is a quantitatively important risk factor in the bond market.

3.5.3 Excess bond return predictability. Under the expectations hypothesis (EH), the expected holding returns from long-term and short-term bonds should be the same (strong form) or should only differ by a constant (weak form). However, even the weak form has been consistently rejected by empirical researchers. For example, Campbell and Shiller (1991); Dai and Singleton (2002); Cochrane and Piazzesi (2005); Bansal and Shaliastovich (2013) all argue that the EH neglects the risks inherent in bonds and provide strong empirical evidence for predictable changes in future excess returns. The first panel of Table 5 compares model-implied distributions for the slope coefficient to the corresponding data estimates. Since the presence of regime switching gives rise to time variations in risk premiums, I only focus on simulation results that allow for actual transition to different regimes (which correspond to the results under "Mix" in previous tables). Here, I also report the posterior interquartile range (5%, 95%), in addition to median values. The first thing to

29 For the CP regime, the increase in the term premium component dominates the downward mean reversion of the average expected short rate component.

Table 6
Model-implied moments: Stock market

	Average stock market moments							
	Data	Model						
		Regime-switching				Fixed-regime		
Maturity		Mix	CA	CP	PA	CA	CP	PA
$E(r_m)$	5.75	4.92	4.78	4.25	4.85	4.94	4.80	4.85
$\sigma(r_m)$	15.48	17.89	11.76	11.94	11.73	11.78	11.80	11.78
$E(pd)$	3.60	3.53	3.47	3.65	3.46	3.53	3.53	3.54
$\sigma(pd)$	37.04	19.05	16.39	17.41	16.04	17.23	16.45	17.16
Average (conditional) stock-bond return correlation								
1-year bond	0.08	0.16	0.71	0.47	−0.72	0.83	0.68	−0.92
2-year bond	0.08	0.18	0.71	0.50	−0.72	0.84	0.68	−0.92
3-year bond	0.09	0.19	0.72	0.52	−0.70	0.84	0.68	−0.91
4-year bond	0.10	0.20	0.73	0.53	−0.70	0.84	0.68	−0.91
5-year bond	0.10	0.20	0.73	0.54	−0.69	0.85	0.68	−0.91

Notes: The appendix provides detailed descriptions of the data. In the bottom panel, the numbers for data are the averages of the conditional stock-bond return correlation reported in Figure A-1.

note is that the model generates very comparable results. The model produces slope coefficients that are significantly negative, lower than unity, and whose absolute magnitudes rise over maturities, like in the data. Another exercise consists of running regressions that predict excess bond returns. Following Cochrane and Piazzesi (2005), I focus on regressing the excess bond return of a n-maturity bond over the one-year bond on a linear combination of forward rates that includes a constant term, a one-year bond yield, and four forwards rates with maturities of two to five years. The model-implied R^2 values (in percentages) from the regression are provided in the second panel of Table 5 and are comparable to (but slightly overshoot) the corresponding data estimates.

3.5.4 Market returns and log price-dividend ratio. Now, I examine the stock market implications of the model in Table 6. The regime-switching model with actual transitions to different regimes (values under "Mix") is able to explain large part of the equity premium $E[r_{m,t+1} - r_{f,t}] + 1/2\sigma(r_m)^2 \approx (0.049 - 0.017) + 0.5 \times 0.1789^2 \approx 5\%$ (the mean of the model-implied risk-free rate is around 1.7%). The standard deviation of the market returns $\sigma(r_m) = 17.89\%$ and price-dividend $\sigma(pd) = 19.05\%$ are comparable to their data counterparts. This feature owes, in part, to the fact that the regime-switching model can accommodate some non-Gaussian features in the data.

3.5.5 Stock-bond return correlation. Since the estimated model is quite successful in explaining several bond and stock market phenomena, I can proceed to examine the interactions between the two. The second panel of Table 6 reports the averages of the model-implied conditional stock-bond return correlation. When transitions to different regimes are allowed (values under "Mix"), the model generates mildly positive stock-bond return correlation.

It is because inflation risks are substantial in the model: The unconditional probability of experiencing countercyclical macroeconomic shocks, indicated by the sum of the probability of the CA and CP regimes, is 0.68. Now I look at the conditional stock-bond return correlation implied by each regime. This experiment is useful because it isolates "within-regime" risks in the stock-bond return correlation. I find that the active monetary policy stance tends to generate stronger positive stock-bond return correlation. My results are consistent with the findings in Campbell, Pflueger, and Viceira (2015) in which they argue that a more aggressive response of the central bank to inflation fluctuations increases the stock-bond return correlation. Through the estimation, I have identified that (while the stance of the monetary policy remained active) the economy faced changes in the covariance between the inflation target and real growth shocks, that is, transition from the countercyclical to the procyclical macroeconomic shock regime, in the late 1990s. It is straightforward to see from Table 1 that as the economy shift towards active monetary policy and experience procyclical macroeconomic shocks, nominal bonds become hedges and nominal bonds behave qualitatively similarly to real bonds. Simulation results in Table 6 confirm that quantitatively the effects are strong enough to generate negative stock-bond return correlation.

3.6 Robustness checks

The model is successful in quantitatively accounting for both new and old bond market stylized facts. The key ingredients of the model include preference for an early resolution of uncertainty, time variation in expected real consumption growth and inflation target, and especially regime switches in the monetary policy action, as well as in the distribution of macroeconomic shocks. With regime switching, through the iterated expectations over the regimes, the model is able to generate negative risk premium and stock-bond return correlation and at the same time produce an upward-sloping nominal yield curve. This is the main improvement over and above the existing works.[30]

Having said that, there are several important caveats that need to be taken into account in this paper. First, the model is estimated with macro and asset pricing data. It would be interesting to know whether the model can do a good job at fitting many asset pricing moments when the model is estimated with macro data only. Second, the model assigns about a 1/3 probability of switching between regimes. A significant chance of encountering high inflation regime is the key mechanism of the model that reconciles a negative stock-bond return correlation with an upward-sloping yield curve. I check the empirical plausibility of the model mechanism using information from inflation option market data.

In addition, I provide a battery of additional robustness checks in the appendix. Specifically, I estimate a prototypical New Keynesian model to

[30] For example, the average slope of the term structure reflects a downward-sloping term structure of −55 bps in Campbell, Pflueger, and Viceira (2015) when they are trying to match the negative stock-bond return correlation.

Table 7
Posterior median estimates: Macro data only

(1)		(2)		(3)		(4)		(5)	
Preference		Consumption		Dividend		Factor shocks		Monetary policy	
δ	0.999	μ_c	0.0016	μ_d	0.0016	$\beta(-)$	−0.75	$\tau_0(A)$	0.0047
							[−1.41, −0.32]		[0.0031, 0.0055]
ψ	2	ρ_c	.99	ϕ	3	$\beta(+)$	0.45	$\tau_0(P)$	0.0045
							[0.02, 0.85]		[0.0031, 0.0054]
γ	8	σ_c	0.0024	σ_d/σ_c	6.25	σ_{xc}	0.00015	$\tau_\pi(A)$	1.19
							[0.00011, 0.00020]		[1.01, 1.95]
						$\sigma_{x\pi}(CA)$	0.00019	$\tau_\pi(P)$	0.97
							[0.00015, 0.00023]		[0.90, 0.99]
						$\sigma_{x\pi}(CP)$	0.00039	$\tau_c(A,P)$	0.56
							[0.00034, 0.00047]		[0.43, 0.66]
						$\sigma_{x\pi}(PA)$	0.00011		
							[0.00007, 0.00015]		
						σ_{xi}	0.0020		
							[0.0013, 0.0027]		

Notes: The estimation results are based on monthly data from 1963:M1 to 2014:M12. A subset of parameters under (1), (2), and (3) is fixed based on Schorfheide, Song, and Yaron (2016). I show the posterior interquartile range (5%, 95%) in brackets.

further provide robustness of the identification of monetary policy regimes. Given the key role played by β in this paper, a more thorough discussion (about the origin and interpretation) is provided. Finally, as recently pointed out by Duffee (2015), standard term structure models (especially these type of long-run risks models with recursive preferences) have known problems of embedding too much inflation risks in the model. I investigate how my model performs along this dimension and demonstrate that allowing for regime switching could be an economically appealing way of modeling inflation dynamics.

3.6.1 Estimation with macroeconomic data only. It is well known that the macroeconomic models are notoriously bad at fitting asset prices. The reader may ask in order for the model to fit the stylized asset pricing moments including the stock-bond return correlation, the paper has to assume highly implausible macroeconomic dynamics. One way to try to provide an answer to such criticism would be to estimate the model using macroeconomic data only, that is, consumption, inflation, and the short interest rate. The parameter estimates are provided in Table 7. There are few notable changes in the estimates: The exogenous correlation coefficients, $\beta(-)$ and $\beta(+)$, are now much smaller in absolute magnitude; the monetary policy loadings on inflation, $\tau_\pi(A)$ and $\tau_\pi(P)$, are closer to one. Table 8 assesses the asset pricing implications of these parameter differentials. Due to space constraints, I only provide the average yield curve, bond risk premiums, and stock-bond return correlation. While different quantitatively, even without the information content of asset prices the model-implied moments are similar qualitatively. To facilitate comparison, the previous results are summarized under "Macro and Asset Data." In sum, the model is still able to generate an upward-sloping yield curve in all regimes and, consistent with before, it can produce negative bond risk premiums and stock-bond return correlation in the PA regime.

Table 8
Model-implied moments: Macro data only

	Average bond yield								
	Data	Regime-switching model							
		Macro and asset data				Macro data only			
Maturity		Mix	CA	CP	PA	Mix	CA	CP	PA
1-year bond	5.26	5.28	5.18	5.33	4.92	5.88	4.82	4.82	5.09
2-year bond	5.47	5.43	5.27	5.40	4.92	5.97	4.85	4.97	5.25
3-year bond	5.65	5.58	5.30	5.47	4.97	5.99	4.86	5.05	5.30
4-year bond	5.81	5.76	5.38	5.52	5.01	6.04	4.90	5.14	5.34
5-year bond	5.92	5.84	5.41	5.63	5.12	6.11	4.96	5.21	5.40
	Average (one-period) bond risk premiums								
1-year bond	—	0.08	0.08	0.07	−0.07	0.12	0.04	0.07	−0.00
2-year bond	—	0.20	0.18	0.18	−0.16	0.25	0.10	0.15	−0.00
3-year bond	—	0.34	0.28	0.30	−0.25	0.37	0.16	0.21	−0.00
4-year bond	—	0.49	0.39	0.43	−0.33	0.49	0.22	0.28	−0.01
5-year bond	—	0.65	0.51	0.55	−0.41	0.60	0.27	0.34	−0.02
	Average (conditional) stock-bond return correlation								
1-year bond	0.08	0.16	0.71	0.47	−0.72	0.31	0.55	0.48	−0.06
2-year bond	0.08	0.18	0.71	0.50	−0.72	0.32	0.57	0.44	−0.03
3-year bond	0.09	0.19	0.72	0.52	−0.70	0.32	0.57	0.42	−0.02
4-year bond	0.10	0.20	0.73	0.53	−0.70	0.30	0.57	0.40	−0.03
5-year bond	0.10	0.20	0.73	0.54	−0.69	0.29	0.57	0.39	−0.04

Notes: The appendix provides detailed descriptions of the data. While the unconditional averages of stock-bond return correlation are small in the data, the conditional stock-bond return correlation fluctuates quite a lot (see Figure A-1).

The paper finds about a 1/3 probability of switching between regimes. It has the interpretation that when agents form their expectations they take into account a 1/3 probability they can go back to the monetary and macro regime of the 1970s. One could say that the 1970s are not around the corner and this probability speaks more about an important limitation of the model to explain the data than to something that is really going on in the data. Figure 4 depicts the smoothed posterior regime probabilities from the model when it is estimated with macro data only (light-blue lines). For ease of comparison, I overlay them with the regime probabilities estimated with macro and asset price data (dark-gray-shaded areas). What is important to note is that the estimated regime probabilities are robust to the exclusion of the asset prices. I find that the low-frequency changes in the data are well captured even when macro data are only used in the estimation. The estimated Markov-chain transition probabilities are very similar to (15). The bottom line is that there is still a 1/3 probability of switching to the CP regime.

3.6.2 Evidence from the inflation option market. Inflation has been low and relatively stable over the past two decades. In fact, we are talking about the possibility of deflation. Given this, one may question the empirical plausibility of the big inflation scare that model requires to match stylized asset pricing facts. One piece of supporting evidence is provided from the inflation option market. Figure 5 shows the time series of probabilities of average inflation

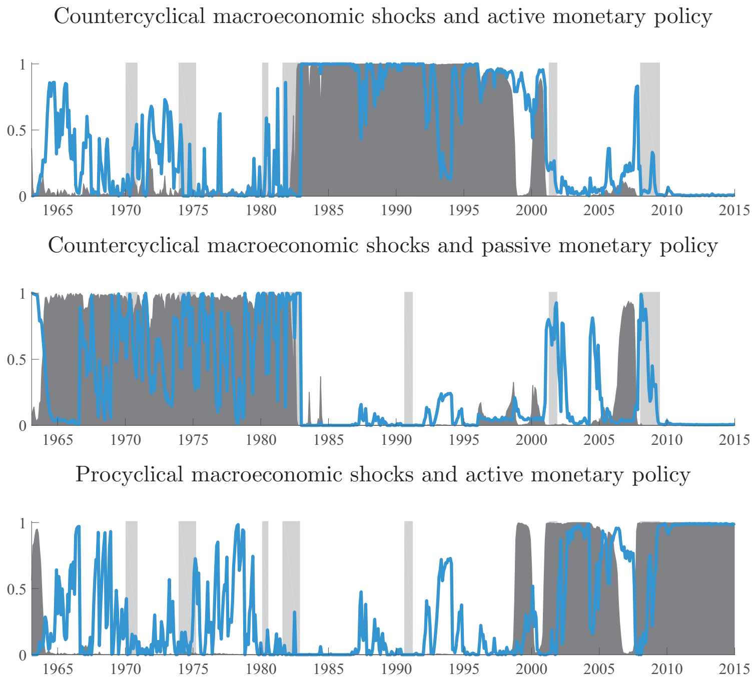

Figure 4
Regime probabilities: Macro data only
The light-blue lines represent posterior medians of regime probabilities estimated with macro data only. The dark-gray-shaded areas represent posterior medians of regime probabilities estimated with macro and asset price data. The light-gray-shaded bars indicate the NBER recession dates.

exceeding 4% per annum over selected horizons, which are computed from inflation caps and floors as in Kitsul and Wright (2013).[31] There are several important takeaways. First, the odds of high inflation exceed 10% on average for all horizons. Second, the odds are greater for longer horizon. At the ten-year horizon, the probabilities reach 20% in some year. The inflation options-implied densities assign relatively large probabilities to high inflation outcomes.[32]

Inflation may not have been much of a concern in recent decades, but the risk is still present. Two potential sources of inflation risks come to my mind. Inflation will be a concern when rising federal debt will ultimately prove unsustainable. Also, debate about a higher inflation target among fed officials

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[31] I thank Jonathan Wright for providing data.

[32] Similar findings can be seen in a rich data set of individual inflation survey forecasts taken from the Survey of Professional Forecasters. The shares of each response group, categorized into $\pi < 0$, $0 \leq \pi < 1$, $1 \leq \pi < 2$, $2 \leq \pi < 3$, $3 \leq \pi < 4$, and $4 \leq \pi$, for the one-quarter-ahead inflation forecasts and one-year-ahead inflation forecasts are provided in Figure E-4. A nonnegligible number of individuals forecast high inflation, $\pi \geq 3\%$.

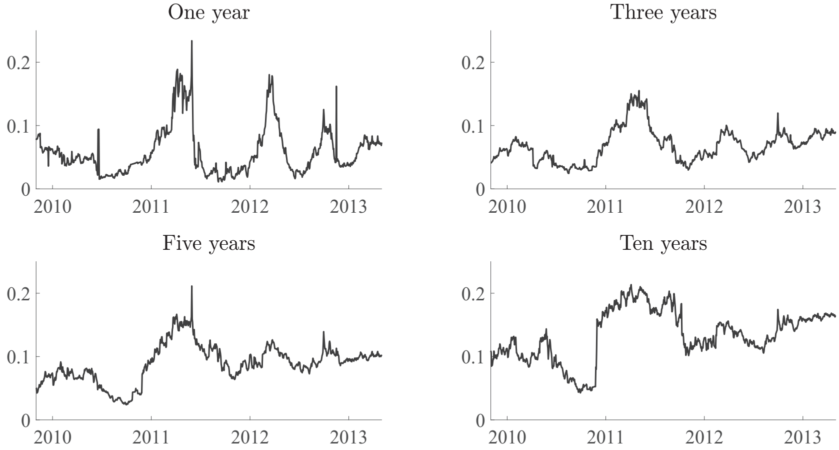

Figure 5
Options-implied probability of high inflation
I reproduce Figure 4 in Kitsul and Wright (2013). This shows the time series of probabilities of average CPI inflation over selected horizons exceeding 4% per annum, which are constructed using inflation caps and floors. The sample period is from October 2009 to April 2013. Kitsul and Wright (2013) use indicative daily quotes on zero-coupon inflation caps at strike prices of 1% to 6%, and on zero-coupon inflation floors at strike prices from 2% to 3%, both in increments of 0.5%. The maturities are one, three, five, and ten years.

could create inflation scares amongst bond traders.[33] All in all, inflation risk should not be ignored.

4. Conclusion and Directions for Future Research

The paper studies the behavior of the nominal U.S. Treasury yield curve and the changing stock-bond return correlations in a model that allows for regime switches in the aggressiveness of monetary policy and in the conditional covariance of macroeconomic shocks. The model follows the long-run risk literature for the real side of the economy and extends it to add the nominal sector and changing regimes. The estimation identifies inflation as countercyclical from the early 1970s through the late 1990s and as procyclical from the late 1990s onward. This is overlaid with the "active" monetary policy regime that dominates most of the sample outside of the 1970s period, which is classified as "passive." The model is used to study the key moments of the yield curve and the correlation between bond-stock returns. It approximately matches the timing during which the stock-bond correlation switches signs from positive to negative in the late 1990s.

A number of other interesting extensions are possible for future research. First, liquidity in bond markets has become a major focus of concern, especially after the financial turmoil. It would be interesting to examine how much of

[33] For references, see Blanchard, Dell'Ariccia, and Mauro (2010); Ball (2014); Williams (2016).

the recent bond market changes are due to liquidity risk. Second, a structural explanation for the underlying economic driving forces behind the change in correlation between growth and inflation can be useful. This requires a model in which monetary neutrality assumption is relaxed, the conditional distributions of structural (demand and supply) shocks vary over time, and, fundamentally, changing propagation mechanism of demand and supply shocks is explored.[34] Third, as suggested by David and Veronesi (2016), it would be interesting to see how the model can account for the international evidence on time-varying stock-bond return correlation.

Appendix A. Data

Monthly consumption data represent per capita series of real consumption expenditures on nondurables and services from the National Income and Product Accounts (NIPA) tables, which are available from the Bureau of Economic Analysis. Aggregate stock market data consist of monthly observations of returns, dividends, and prices of the CRSP value-weighted portfolio of all stocks traded on the NYSE, AMEX, and NASDAQ. Price and dividend series are constructed on a per share basis, like in Campbell and Shiller (1988b); Hodrick (1992). Market data are converted to real data using the consumer price index (CPI) from the Bureau of Labor Statistics. Growth rates of consumption and dividends are constructed by taking the first difference of the corresponding log series. Inflation represents the log difference of the CPI. Monthly observations of U.S. Treasury bonds with maturities at one to five years are from CRSP. The time series spans the monthly data from 1963:M1 to 2014:M12.

Table A-1
Data (annualized)

Consumption		Inflation	
$E(\Delta c)$	1.96	$E(\pi)$	3.71
$\sigma(\Delta c)$	1.16	$\sigma(\pi)$	1.10
$AC(\Delta c)$	−0.16	$AC(\pi)$	0.62
$corr(\Delta c, \pi)$	−0.18		
Log price-dividend ratio		**Stock market returns**	
$E(pd)$	3.60	$E(r_m)$	5.75
$\sigma(pd)$	0.37	$\sigma(r_m)$	15.48
Bond yields		**Stock-bond return correlation**	
$E(y_1^\$)$	5.26	$E(corr(r_m, y_1^\$))$	0.08
$E(y_2^\$)$	5.47	$E(corr(r_m, y_2^\$))$	0.08
$E(y_3^\$)$	5.65	$E(corr(r_m, y_3^\$))$	0.09
$E(y_4^\$)$	5.81	$E(corr(r_m, y_4^\$))$	0.10
$E(y_5^\$)$	5.92	$E(corr(r_m, y_5^\$))$	0.10

Notes: I use daily stock market returns and k-year bond returns to compute the realized conditional correlation, $corr_t(r_{m,t+1}, r^\$_{ky,t+1})$, where $k=1,...,5$. The sample ranges from 1963:M1 to 2014:M12.

34 The recent work of Gourio and Ngo (2016) is interesting because it has the potential to address the issue.

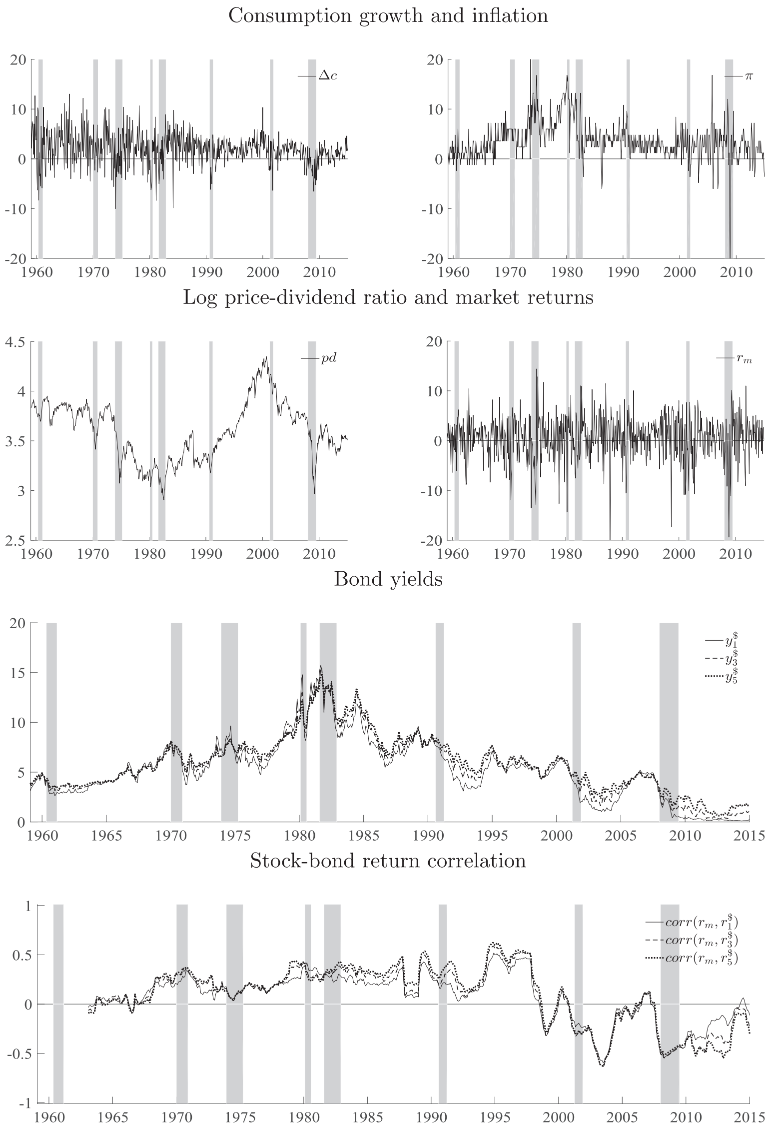

Figure A-1
Data (annualized)
All data are annualized. The light-gray-shaded bars indicate the NBER recession dates.

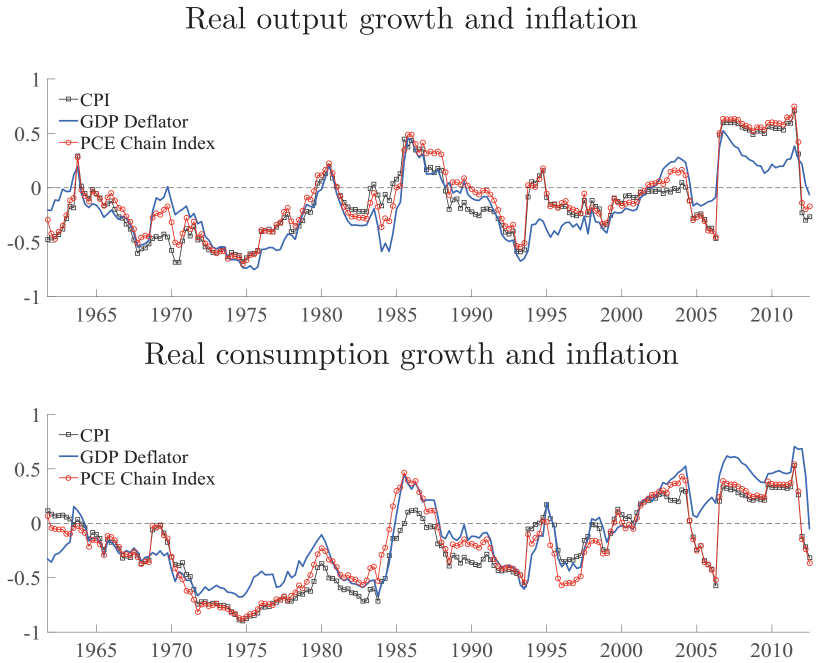

Figure A-2
Time-varying real growth and inflation correlation
I provide rolling sample correlation. The window size is five years. All data are available in quarterly frequency. Three different inflation measures are considered.

Appendix B. Piazzesi and Schneider Revisited

Following Piazzesi and Schneider (2006), I assume that the vector of inflation and consumption growth has the following state space representation:

$$z_t = \mu(S_t) + x_{t-1} + \varepsilon_t, \quad z_t = [\pi_t, \Delta c_t]'$$

$$x_t = \phi(S_t)x_{t-1} + \phi(S_t)K(S_t)\varepsilon_t, \quad \varepsilon_t \sim N(0, \Omega(S_t)). \tag{B1}$$

The state vector x_t is two dimensional and contains expected inflation and consumption; ϕ is the 2×2 autoregressive matrix; and K is the 2×2 gain matrix, where

$$\phi = \begin{bmatrix} \phi_1 & \phi_{12} \\ \phi_{21} & \phi_2 \end{bmatrix}, \quad K = \begin{bmatrix} k_1 & k_{12} \\ k_{21} & k_2 \end{bmatrix}, \quad \Omega = \begin{bmatrix} \Omega_1 & \Omega_{12} \\ \Omega_{12} & \Omega_2 \end{bmatrix}.$$

S_t denotes the regime indicator variable $S_t \in \{1,2\}$. Using the Bayesian method, I estimate this two-state regime-switching system with data for consumption and inflation. Table B-2 provides the details of parameter prior and posterior distributions. Because the complete estimation information in the tables can be difficult to absorb fully, I briefly present aspects of the results in a more revealing way. The parameters to be estimated are those in the transition equation ϕ, K and those in the covariance matrix Ω. Hence, I simply display the estimated transition equation and the estimated Ω matrices.

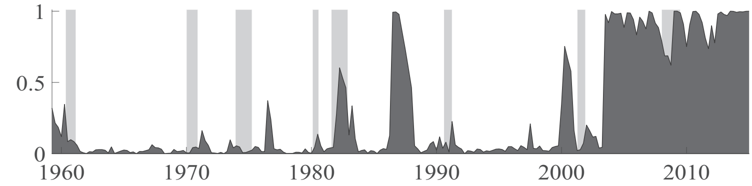

Figure B-3
Probabilities of regime 2 in (B1)
The dark-gray-shaded areas represent posterior medians of regime probabilities. The light-gray-shaded bars indicate the NBER recession dates.

1. Regime 1,

$$x_t = \begin{bmatrix} 0.987 & 0.072 \\ -0.063 & 0.632 \end{bmatrix} x_{t-1} + \begin{bmatrix} 0.606 & 0.323 \\ -0.367 & 0.284 \end{bmatrix} \epsilon_t$$

$$\epsilon_t \sim N\left(0, \begin{bmatrix} 1.195 & -0.086 \\ -0.086 & 2.198 \end{bmatrix}\right), \quad var(\phi K \epsilon_t) = \begin{bmatrix} 0.634 & -0.069 \\ -0.069 & 0.284 \end{bmatrix}.$$

2. Regime 2,

$$x_t = \begin{bmatrix} 0.149 & 0.168 \\ 0.250 & 0.589 \end{bmatrix} x_{t-1} + \begin{bmatrix} 0.107 & 0.119 \\ 0.145 & 0.405 \end{bmatrix} \epsilon_t$$

$$\epsilon_t \sim N\left(0, \begin{bmatrix} 9.224 & -0.005 \\ -0.005 & 1.801 \end{bmatrix}\right), \quad var(\phi K \epsilon_t) = \begin{bmatrix} 0.132 & 0.231 \\ 0.231 & 0.490 \end{bmatrix}.$$

Figure B-3 provides the smoothed posterior regime probabilities. The estimation allows us to split the sample into two regimes: the economy was in regime 1 from the 1960s through the late 1990s and in regime 2 from late 1990s onward.[35]

To understand the key properties of the estimated dynamics, Table B-3 reports the sample moments implied by the posterior estimates in Table B-2. Here, I mention a few of many noteworthy aspects of the results. There is a dramatic change in inflation dynamics. I find a large decline in inflation persistence. Also, the correlation between inflation and consumption growth changed from negative to positive.

[35] With the exception of 1986 and 1987. This is consistent with Figure A-2.

Table B-2
Posterior estimates

				Regime 1			Regime 2		
		Prior		Posterior			Posterior		
	Distr.	20%	80%	20%	50%	80%	20%	50%	80%
ϕ_1	N^T	[−.35	.99]	0.976	0.987	0.995	0.061	0.149	0.282
ϕ_{12}	N	[−.82	.82]	0.016	0.072	0.138	0.027	0.168	0.301
ϕ_{21}	N	[−.82	.82]	−0.103	−0.063	−0.030	0.138	0.250	0.341
ϕ_2	N^T	[−.35	.99]	0.521	0.632	0.729	0.392	0.589	0.732
k_1	N	[.15	1.81]	0.556	0.652	0.758	0.457	0.850	1.147
k_{12}	N	[−.82	.82]	0.225	0.292	0.366	−0.316	0.045	0.422
k_{21}	N	[−.82	.82]	−0.691	−0.515	−0.360	−0.380	−0.114	0.125
k_2	N	[.15	1.81]	0.366	0.478	0.637	0.425	0.669	0.983
Ω_1	IG	[0.80	11.78]	1.035	1.195	1.390	7.618	9.224	10.946
Ω_{12}	N	[−.82	.82]	−0.162	−0.086	−0.009	−0.094	−0.005	0.078
Ω_2	IG	[0.80	11.78]	2.009	2.198	2.445	1.367	1.801	2.485
μ_π	N	[−5.34	11.26]	0.224	3.901	8.467	3.413	5.604	6.469
$\mu_{\Delta c}$	N	[−5.34	11.26]	1.912	2.090	2.286	1.434	1.745	2.035

Notes: The estimation results are based on (annualized) quarterly consumption growth data and inflation data from 1959:Q1 to 2014:Q4. N, N^T, and IG are normal, truncated (outside of the interval $(-1,1)$) normal, and inverse gamma distributions, respectively.

Table B-3
Model-implied moments: Macroeconomic aggregates

	Regime 1			Regime 2		
	05%	50%	95%	05%	50%	95%
$E(\Delta c)$	1.44	2.19	2.89	0.65	1.53	2.30
$\sigma(\Delta c)$	1.54	1.80	2.13	1.28	1.62	2.26
$AC(\Delta c)$	0.19	0.37	0.55	0.21	0.55	0.86
$corr(\Delta c,\pi)$	−0.54	−0.32	−0.11	−0.10	0.05	0.22
$E(\pi)$	0.65	4.26	7.83	1.85	2.20	2.55
$\sigma(\pi)$	1.97	2.89	4.72	2.05	2.49	3.19
$AC(\pi)$	0.78	0.90	0.96	−0.08	0.09	0.28

Notes: I report the percentiles of the posterior predictive distribution for various sample moments based on simulations from the posterior distribution of the same length as the data.

Appendix C. Asset Pricing Solution of a Regime-Switching Model

C.1 Derivation of Approximate Analytical Solutions

The Euler equation for the economy is

$$1 = E_t\left[\exp\left(m_{t+1} + r_{k,t+1}\right)\right], \quad k \in \{c,m\}, \tag{C2}$$

where $m_{t+1} = \theta \log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)r_{c,t+1}$ is the log stochastic discount factor, $r_{c,t+1}$ is the log return on the consumption claim, and $r_{m,t+1}$ is the log market return. All returns are given by the approximation of Campbell and Shiller (1988a):

$$r_{c,t+1} = \kappa_{0,c} + \kappa_{1,c} z_{c,t+1} - z_{c,t} + g_{c,t+1},$$

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1}. \tag{C3}$$

The first-order condition of the agent's expected utility maximization problem yields the Euler equations

$$\mathbb{E}_t\left[\exp\left(m_{t+1} + r_{k,t+1}\right)\right] = 1, \quad k \in \{c,m\}, \quad \text{Real Assets}, \tag{C4}$$

$$p^{\$}_{n,t} = \log \mathbb{E}_t[\exp(m_{t+1} - \pi_{t+1} + p^{\$}_{n-1,t+1})], \quad \text{Nominal Assets}, \tag{C5}$$

where $r_{c,t+1}$ is the log return on the consumption claim, $r_{m,t+1}$ is the log market return, and $p_{n,t}^{\$}$ is the nominal n-maturity log bond price. The solutions to (C4) and (C5) depend on the joint dynamics of consumption, dividend growth, and inflation.

Asset prices are determined from the approximate analytical solution described by Bansal and Zhou (2002). Let I_t denote the current information set $\{S_t, X_t\}$ and define $I_{t+1}=I_t \cup \{S_{t+1}\}$, which includes information regarding S_{t+1} in addition to I_t. The derivation of (C4) follows Bansal and Zhou (2002), who repeatedly use the law of iterated expectations. For example, real asset returns are determined by

$$
\begin{aligned}
1 &= \mathbb{E}\Big(\mathbb{E}\big[\exp\big(m_{t+1}+r_{m,t+1}\big) \mid I_{t+1}\big] \mid I_t\Big)\\
&= \sum_{j=1}^{3} \Pi_{ij}\mathbb{E}\Big(\exp\big(m_{t+1}+r_{m,t+1}\big) \mid S_{t+1}=j, X_t\Big)\\
0 &= \sum_{j=1}^{3} \Pi_{ij} \underbrace{\Big(\mathbb{E}\big[m_{t+1}+r_{m,t+1} \mid S_{t+1}=j, X_t\big]+\frac{1}{2}\mathbb{V}\big[m_{t+1}+r_{m,t+1} \mid S_{t+1}=j, X_t\big]\Big)}_{B}.
\end{aligned}
$$

The first line uses the law of iterated expectations; the second line uses the definition of Markov chain; and the third line applies log-linearization (i.e., $\exp(B)-1 \approx B$) and a log-normality assumption. The derivation of (C5) can be done analogously.

C.2 Markov Chain

The model parameters evolve according to a three-state Markov chain $S_t \in \{1,2,3\}$. I define a Markov transition probability matrix by

$$
\Pi = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}
$$

where $\sum_{j=1}^{3} \Pi_{ij} = 1$ for $\forall i \in \{1,2,3\}$.

C.3 Real Endowments

With regime-switching coefficients, the joint consumption and dividend dynamics are

$$
\begin{aligned}
G_{t+1} &= \mu + \varphi X_t + \Sigma \eta_{t+1}, \quad \eta_t \sim N(0, I),\\
X_{t+1} &= \Phi(S_{t+1}) X_t + \Omega(S_{t+1}) \Sigma_x(S_{t+1}) \eta_{x,t+1}, \quad \eta_{x,t} \sim N(0, I),
\end{aligned}
$$

where $G_t = [\Delta c_t, \Delta d_t]'$, $\mu = [\mu_c, \mu_d]'$, $\eta_t = \big[\eta_{c,t}, \eta_{d,t}\big]'$, $X_t = \big[x_{c,t}, x_{\pi,t}, x_{i,t}\big]'$, $\eta_{x,t} = \big[\eta_{xc,t}, \eta_{x\pi,t}, \eta_{xi,t}\big]'$ and

$$
\varphi = \begin{bmatrix} 1 & 0 & 0 \\ \phi & 0 & 0 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_c & 0 \\ 0 & \sigma_d \end{bmatrix},
$$

$$
\Phi = \begin{bmatrix} \rho_c & 0 & 0 \\ 0 & \rho_\pi & 0 \\ 0 & 0 & \rho_i \end{bmatrix}, \quad \Omega = \begin{bmatrix} 1 & 0 & 0 \\ \beta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Sigma_x = \begin{bmatrix} \sigma_{xc} & 0 & 0 \\ 0 & \sigma_{x\pi} & 0 \\ 0 & 0 & \sigma_{xi} \end{bmatrix}.
$$

C.4 Real Consumption Claim

If the conjectured solution to log price-consumption ratio is

$$z_{c,t}=A_0(S_t)+A_1(S_t)X_t,$$

then the return on the consumption claim can be written as

$$r_{c,t+1}=\kappa_0+\mu_c+\kappa_1 A_0(S_{t+1})-A_0(S_t)+\Big(e_1+\kappa_1 A_1(S_{t+1})\Phi(S_{t+1})-A_1(S_t)\Big)X_t$$

$$+\kappa_1 A_1(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1}+e_1\Sigma\eta_{t+1}.$$

The solutions for the A's that describe the dynamics of the price-consumption ratio are determined from

$$E_t(m_{t+1}+r_{c,t+1})+\frac{1}{2}Var_t(m_{t+1}+r_{c,t+1})=0.$$

$$\begin{bmatrix}A_1(1)'\\A_1(2)'\\A_1(3)'\end{bmatrix}=\begin{bmatrix}I-p_{11}\kappa_1\Phi(1) & -p_{12}\kappa_1\Phi(2) & -p_{13}\kappa_1\Phi(3)\\-p_{21}\kappa_1\Phi(1) & I-p_{22}\kappa_1\Phi(2) & -p_{23}\kappa_1\Phi(3)\\-p_{31}\kappa_1\Phi(1) & -p_{32}\kappa_1\Phi(2) & I-p_{33}\kappa_1\Phi(3)\end{bmatrix}^{-1}(1-\frac{1}{\psi})\begin{bmatrix}e_1'\\e_1'\\e_1'\end{bmatrix},$$

$$\begin{bmatrix}A_0(1)\\A_0(2)\\A_0(3)\end{bmatrix}=(I-\kappa_1\Pi)^{-1}\Pi\begin{bmatrix}\log\delta+\kappa_0+(1-\frac{1}{\psi})\mu_c+\frac{\theta}{2}(1-\frac{1}{\psi})^2 e_1\Sigma\Sigma'e_1'+\frac{\theta}{2}\Psi(1)\Psi(1)'\\\log\delta+\kappa_0+(1-\frac{1}{\psi})\mu_c+\frac{\theta}{2}(1-\frac{1}{\psi})^2 e_1\Sigma\Sigma'e_1'+\frac{\theta}{2}\Psi(2)\Psi(2)'\\\log\delta+\kappa_0+(1-\frac{1}{\psi})\mu_c+\frac{\theta}{2}(1-\frac{1}{\psi})^2 e_1\Sigma\Sigma'e_1'+\frac{\theta}{2}\Psi(3)\Psi(3)'\end{bmatrix},$$

$$\Psi(S_t)=\kappa_1 A_1(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1}).$$

The log pricing kernel is

$$m_{t+1}=\theta\log\delta+(\theta-1)\big(\kappa_0+\kappa_1 A_0(S_{t+1})-A_0(S_t)\big)-\gamma\mu_c$$

$$-\frac{1}{\psi}e_1X_t+(\theta-1)\Big((1-\frac{1}{\psi})e_1+\kappa_1 A_1(S_{t+1})\Phi(S_{t+1})-A_1(S_t)\Big)X_t$$

$$-\gamma e_1\Sigma\eta_{t+1}+(\theta-1)\kappa_1 A_1(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1}.$$

C.5 Real Dividend Claim/Market Return

Analogously, the conjectured solution to log price-dividend ratio is

$$z_{m,t}=A_{0,m}(S_t)+A_{1,m}(S_t)X_t,$$

and the return on the dividend claim can be written as

$$r_{m,t+1}=\kappa_{0,m}+\mu_d+\kappa_{1,m}A_{0,m}(S_{t+1})-A_{0,m}(S_t)+\Big(\phi e_1+\kappa_{1,m}A_{1,m}(S_{t+1})\Phi(S_{t+1})-A_{1,m}(S_t)\Big)X_t$$

$$+\kappa_{1,m}A_{1,m}(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1}+e_2\Sigma\eta_{t+1}.$$

The solutions for A_ms are

$$\begin{bmatrix}A_{1,m}(1)'\\A_{1,m}(2)'\\A_{1,m}(3)'\end{bmatrix}=\begin{bmatrix}I-p_{11}\kappa_{1,m}\Phi(1) & -p_{12}\kappa_{1,m}\Phi(2) & -p_{13}\kappa_{1,m}\Phi(3)\\-p_{21}\kappa_{1,m}\Phi(1) & I-p_{22}\kappa_{1,m}\Phi(2) & -p_{23}\kappa_{1,m}\Phi(3)\\-p_{31}\kappa_{1,m}\Phi(1) & -p_{32}\kappa_{1,m}\Phi(2) & I-p_{33}\kappa_{1,m}\Phi(3)\end{bmatrix}^{-1}(\phi-\frac{1}{\psi})\begin{bmatrix}e_1'\\e_1'\\e_1'\end{bmatrix},$$

$$\begin{bmatrix}A_{0,m}(1)\\A_{0,m}(2)\\A_{0,m}(3)\end{bmatrix}=(I-\kappa_{1,m}\Pi)^{-1}\left(\Pi\begin{bmatrix}(\theta-1)\kappa_1A_0(1)+\frac{1}{2}\Psi_m(1)\Psi_m(1)'\\(\theta-1)\kappa_1A_0(2)+\frac{1}{2}\Psi_m(2)\Psi_m(2)'\\(\theta-1)\kappa_1A_0(3)+\frac{1}{2}\Psi_m(3)\Psi_m(3)'\end{bmatrix}+\begin{bmatrix}\Xi_m(1)\\\Xi_m(2)\\\Xi_m(3)\end{bmatrix}\right),$$

$$\Xi_m(S_t)=\theta\log\delta+(\theta-1)(\kappa_0-A_0(S_t))-\gamma\mu_c+\kappa_{0,m}+\mu_d+\frac{1}{2}\Big(\gamma^2e_1\Sigma\Sigma'e_1'+e_2\Sigma\Sigma'e_2'\Big),$$

$$\Psi_m(S_t)=\Big((\theta-1)\kappa_1A_1(S_{t+1})+\kappa_{1,m}A_{1,m}(S_{t+1})\Big)\Omega(S_t)\Sigma_x(S_t).$$

C.6 Linearization Parameters

Let $\bar{p}_j = \sum_{i \in 3} \bar{p}_i \Pi_{ij}$. For any asset, the linearization parameters are endogenously determined by the following system of equations:

$$\bar{z}_i = \sum_{j=1}^{3} \bar{p}_j A_{0,i}(j),$$

$$\kappa_{1,i} = \frac{\exp(\bar{z}_i)}{1+\exp(\bar{z}_i)},$$

$$\kappa_{0,i} = \log(1+\exp(\bar{z}_i)) - \kappa_{1,i}\bar{z}_i.$$

The solution is numerically determined by iteration until reaching a fixed point of $\bar{z}_i$ for $i \in \{c, m\}$.

C.7 Real Bond Prices

The real n-maturity log bond price satisfies

$$p_{n,t}(S_t) = C_{n,0}(S_t) + C_{n,1}(S_t)X_t,$$

$$= E_t(p_{n-1,t+1}(S_{t+1}) + m_{t+1}) + \frac{1}{2} Var_t(p_{n-1,t+1}(S_{t+1}) + m_{t+1}),$$

where the bond loadings follow the recursions

$$\begin{bmatrix} C_{n,1}(1,:) \\ C_{n,1}(2,:) \\ C_{n,1}(3,:) \end{bmatrix} = \Pi \begin{bmatrix} C_{n-1,1}(1,:)\Phi(1) \\ C_{n-1,1}(2,:)\Phi(2) \\ C_{n-1,1}(3,:)\Phi(3) \end{bmatrix} - \frac{1}{\psi} \begin{bmatrix} e_1 \\ e_1 \\ e_1 \end{bmatrix}$$

$$\begin{bmatrix} C_{n,0}(1) \\ C_{n,0}(2) \\ C_{n,0}(3) \end{bmatrix} = \Pi \begin{bmatrix} C_{n-1,0}(1) + (\theta-1)\kappa_1 A_0(1) + \frac{1}{2}\Psi_{n-1,c}(1)\Psi_{n-1,c}(1)' \\ C_{n-1,0}(2) + (\theta-1)\kappa_1 A_0(2) + \frac{1}{2}\Psi_{n-1,c}(2)\Psi_{n-1,c}(2)' \\ C_{n-1,0}(3) + (\theta-1)\kappa_1 A_0(3) + \frac{1}{2}\Psi_{n-1,c}(3)\Psi_{n-1,c}(3)' \end{bmatrix} + \begin{bmatrix} \Xi_c(1) \\ \Xi_c(2) \\ \Xi_c(3) \end{bmatrix}$$

$$\Xi_c(S_t) = \theta \log\delta + (\theta-1)(\kappa_0 - A_0(S_t)) - \gamma\mu_c + \frac{1}{2}\gamma^2 e_1 \Sigma \Sigma' e_1'$$

$$\Psi_{n-1,c}(S_t) = \left\{ C_{n-1,1}(S_t) + (\theta-1)\kappa_1 A_1(S_t) \right\} \Omega(S_t) \Sigma_x(S_t).$$

The loadings on real bond yields are

$$B_{n,0} = -\frac{1}{n} C_{n,0}, \quad B_{n,1} = -\frac{1}{n} C_{n,1}.$$

The log return to holding a n-maturity real bond from t to $t+1$ is

$$r_{n,t+1} = C_{n-1,0}(S_{t+1}) - C_{n,0}(S_t) + \Big(C_{n-1,1}(S_{t+1})\Phi(S_{t+1}) - C_{n,1}(S_t) \Big) X_t$$

$$+ C_{n-1,1}(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1}.$$

The log return to holding a n-maturity real bond from t to $t+1$ in excess of the log return to a one-period real bond is

$$rx_{n,t+1} = C_{n-1,0}(S_{t+1}) - C_{n,0}(S_t) + C_{1,0}(S_t) + \Big(C_{n-1,1}(S_{t+1})\Phi(S_{t+1}) - C_{n,1}(S_t) + C_{1,1}(S_t) \Big) X_t$$

$$+ C_{n-1,1}(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1}.$$

The one-period expected excess return on real bonds can be written in the following form:

$$E(rx_{n,t+1}|S_t=k)+\frac{1}{2}Var(rx_{n,t+1}|S_t=k)=-Cov(m_{t+1},rx_{n,t+1}|S_t=k)$$

$$=-\Pi(k,:)\times\left(\begin{bmatrix}\Lambda_1(k,1)\Lambda_2(k,1)+\Lambda_3(k,1)\\ \Lambda_1(k,2)\Lambda_2(k,2)+\Lambda_3(k,2)\\ \Lambda_1(k,3)\Lambda_2(k,3)+\Lambda_3(k,3)\end{bmatrix}\right)$$

$$\approx-\Pi(k,:)\times\begin{bmatrix}\Lambda_3(k,1)\\ \Lambda_3(k,2)\\ \Lambda_3(k,3)\end{bmatrix},$$

where the approximation is exact in a fixed-regime economy and

$$\Lambda_1(k,j)=K_t'C(j)'-\Pi(k,:)\begin{bmatrix}K_t'C(1)'\\ K_t'C(2)'\\ K_t'C(3)'\end{bmatrix}$$

$$\Lambda_2(k,j)=M(j)K_t-\Pi(k,:)\begin{bmatrix}M(1)K_t\\ M(2)K_t\\ M(3)K_t\end{bmatrix}$$

$$\Lambda_3(k,j)=(\theta-1)\kappa_1A_1(j)\Omega(j)\Sigma_x(j)\Sigma_x(j)'\Omega_x(j)'C_{n-1,1}(j)'$$

and

$$M(j)=\begin{bmatrix}(\theta-1)\kappa_1A_0(j) & (\theta-1)\kappa_1A_1(j)\Phi(j)\end{bmatrix}$$

$$C(j)=\begin{bmatrix}C_{n-1,0}(j) & C_{n-1,1}(j)\Phi(j)\end{bmatrix}$$

$$K_t=\begin{bmatrix}1\\ X_t\end{bmatrix}.$$

Intuitively, I consider the limiting case in which the regime is fixed. The one-period expected excess return is

$$E_t(rx_{n,t+1})+\frac{1}{2}Var_t(rx_{n,t+1})=-(\theta-1)\kappa_1A_1\Omega\Sigma_x\Sigma_x'\Omega_x'C_{n-1,1}' \tag{C6}$$

$$=-(\gamma-\frac{1}{\psi})\left(\frac{\kappa_1/\psi}{1-\kappa_1\rho_c}\right)\left(\frac{1-\rho_c^{n-1}}{1-\rho_c}\right)$$

where

$$-(\theta-1)\kappa_1A_1=\begin{bmatrix}(\gamma-\frac{1}{\psi})\frac{\kappa_1}{1-\kappa_1\rho_c}, & 0\end{bmatrix}$$

$$\Omega\Sigma_x\Sigma_x'\Omega_x'=\begin{bmatrix}\sigma_{xc}^2 & \beta\sigma_{xc}^2\\ \beta\sigma_{xc}^2 & \beta^2\sigma_{xc}^2+\sigma_{x\pi}^2\end{bmatrix}$$

$$C_{n-1,1}=\begin{bmatrix}-\frac{1}{\psi}\left(\frac{1-\rho_c^{n-1}}{1-\rho_c}\right), & 0\end{bmatrix}.$$

We can conveniently express the sign of the one-period expected excess return as

$$sign\left(E_t(rx_{n,t+1})+\frac{1}{2}Var_t(rx_{n,t+1})\right)=-sign(\gamma-\frac{1}{\psi}).$$

C.8 Endogenous Inflation Process under a Regime-Switching Monetary Policy Rule

The inflation target augmented monetary policy rule is

$$i_t = \tau_0(S_t) + \tau_c(S_t) x_{c,t} + \tau_\pi(S_t)(\pi_t - \Gamma_0(S_t) - x_{\pi,t}) + x_{\pi,t} + x_{i,t}, \tag{C7}$$

and the conjectured solution for inflation process is

$$\pi_t = \Gamma_0(S_t) + \underbrace{\left[\Gamma_{1,c}(S_t), \quad \Gamma_{1,\pi}(S_t), \quad \Gamma_{1,i}(S_t)\right]}_{\Gamma_1(S_t)} X_t. \tag{C8}$$

Combining Equations (C7) and (C8), I rewrite the monetary policy rule as

$$i_t = \tau_0(S_t) + \left[\tau_c(S_t) + \tau_\pi(S_t)\Gamma_{1,c}(S_t), \quad 1 - \tau_\pi(S_t) + \tau_\pi(S_t)\Gamma_{1,\pi}(S_t), \quad 1 + \tau_\pi(S_t)\Gamma_{1,i}(S_t)\right] X_t. \tag{C9}$$

Setting Equation (C9) equal to

$$i_t = -E_t(m_{t+1} - \pi_{t+1}) - \frac{1}{2} Var_t(m_{t+1} - \pi_{t+1}),$$

we can solve for

$$\begin{bmatrix} \Gamma_{1,c}(1) \\ \Gamma_{1,c}(2) \\ \Gamma_{1,c}(3) \end{bmatrix} = \left(\begin{bmatrix} \tau_\pi(1) & 0 & 0 \\ 0 & \tau_\pi(2) & 0 \\ 0 & 0 & \tau_\pi(3) \end{bmatrix} - \Pi \begin{bmatrix} \rho_c(1) & 0 & 0 \\ 0 & \rho_c(2) & 0 \\ 0 & 0 & \rho_c(3) \end{bmatrix} \right)^{-1} \begin{bmatrix} \frac{1}{\psi} - \tau_c(1) \\ \frac{1}{\psi} - \tau_c(2) \\ \frac{1}{\psi} - \tau_c(3) \end{bmatrix},$$

$$\begin{bmatrix} \Gamma_{1,\pi}(1) \\ \Gamma_{1,\pi}(2) \\ \Gamma_{1,\pi}(3) \end{bmatrix} = \left(\begin{bmatrix} \tau_\pi(1) & 0 & 0 \\ 0 & \tau_\pi(2) & 0 \\ 0 & 0 & \tau_\pi(3) \end{bmatrix} - \Pi \begin{bmatrix} \rho_\pi(1) & 0 & 0 \\ 0 & \rho_\pi(2) & 0 \\ 0 & 0 & \rho_\pi(3) \end{bmatrix} \right)^{-1} \begin{bmatrix} \tau_\pi(1) - 1 \\ \tau_\pi(2) - 1 \\ \tau_\pi(3) - 1 \end{bmatrix},$$

$$\begin{bmatrix} \Gamma_{1,i}(1) \\ \Gamma_{1,i}(2) \\ \Gamma_{1,i}(3) \end{bmatrix} = -\left(\begin{bmatrix} \tau_\pi(1) & 0 & 0 \\ 0 & \tau_\pi(2) & 0 \\ 0 & 0 & \tau_\pi(3) \end{bmatrix} - \Pi \begin{bmatrix} \rho_i(1) & 0 & 0 \\ 0 & \rho_i(2) & 0 \\ 0 & 0 & \rho_i(3) \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$

and the constant

$$\begin{bmatrix} \Gamma_0(1) \\ \Gamma_0(2) \\ \Gamma_0(3) \end{bmatrix} = \Pi^{-1} \left(\begin{bmatrix} \Psi_\pi(1) \\ \Psi_\pi(2) \\ \Psi_\pi(3) \end{bmatrix} \right) + \begin{bmatrix} \Xi_\pi(1) \\ \Xi_\pi(2) \\ \Xi_\pi(3) \end{bmatrix},$$

where

$$\begin{aligned} \Xi_\pi(S_t) = & (\theta - 1)\kappa_1 A_0(S_t) \\ & + \frac{1}{2} \left\{ \left((\theta - 1)\kappa_1 A_1(S_t) - \Gamma_1(S_t) \right) \Omega(S_t) \Sigma_x(S_t) \right\} \\ & \left\{ \left((\theta - 1)\kappa_1 A_1(S_t) - \Gamma_1(S_t) \right) \Omega(S_t) \Sigma_x(S_t) \right\}', \end{aligned}$$

$$\Psi_\pi(S_t) = \tau_0(S_t) + \left(\theta \log \delta + (\theta - 1)\kappa_0 - \gamma \mu_c + \frac{\gamma^2}{2} e_1 \Sigma \Sigma' e_1' \right) - (\theta - 1) A_0(S_t).$$

C.9 Nominal Bond Prices

The log nominal pricing kernel is

$$
\begin{aligned}
m^{\$}_{t+1} &= m_{t+1} - \pi_{t+1} \\
&= \theta \log \delta + (\theta - 1)\big(\kappa_0 + \kappa_1 A_0(S_{t+1}) - A_0(S_t)\big) - \gamma \mu_c - \Gamma_0(S_{t+1}) \\
&\quad - \left(\frac{1}{\psi} e_1 + \Gamma_1(S_{t+1})\Phi(S_{t+1})\right) X_t + (\theta - 1)\left((1 - \frac{1}{\psi}) e_1 + \kappa_1 A_1(S_{t+1})\Phi(S_{t+1}) - A_1(S_t)\right) X_t \\
&\quad - \gamma e_1 \Sigma \eta_{t+1} + \Big((\theta - 1)\kappa_1 A_1(S_{t+1}) - \Gamma_1(S_{t+1})\Big) \Omega(S_{t+1}) \Sigma_x(S_{t+1}) \eta_{x,t+1}.
\end{aligned}
$$

The nominal n-maturity log bond price satisfies

$$
\begin{aligned}
p^{\$}_{n,t}(S_t) &= C^{\$}_{n,0}(S_t) + C^{\$}_{n,1}(S_t) X_t, \\
&= E_t(p^{\$}_{n-1,t+1}(S_{t+1}) + m_{t+1} - \pi_{t+1}) + \frac{1}{2} Var_t(p^{\$}_{n-1,t+1}(S_{t+1}) + m_{t+1} - \pi_{t+1}),
\end{aligned}
$$

where the bond loadings follow the recursions

$$
\begin{bmatrix} C^{\$}_{n,1}(1,:) \\ C^{\$}_{n,1}(2,:) \\ C^{\$}_{n,1}(3,:) \end{bmatrix} = \Pi \begin{bmatrix} \{C^{\$}_{n-1,1}(1,:) - \Gamma_1(1)\}\Phi(1) \\ \{C^{\$}_{n-1,1}(2,:) - \Gamma_1(2)\}\Phi(2) \\ \{C^{\$}_{n-1,1}(3,:) - \Gamma_1(3)\}\Phi(3) \end{bmatrix} - \frac{1}{\psi} \begin{bmatrix} e_1 \\ e_1 \\ e_1 \end{bmatrix}
$$

$$
\begin{bmatrix} C^{\$}_{n,0}(1) \\ C^{\$}_{n,0}(2) \\ C^{\$}_{n,0}(3) \end{bmatrix} = \Pi \begin{bmatrix} C^{\$}_{n-1,0}(1) - \Gamma_0(1) + (\theta-1)\kappa_1 A_0(1) + \frac{1}{2}\Psi_{n-1,c}(1)\Psi_{n-1,c}(1)' \\ C^{\$}_{n-1,0}(2) - \Gamma_0(2) + (\theta-1)\kappa_1 A_0(2) + \frac{1}{2}\Psi_{n-1,c}(2)\Psi_{n-1,c}(2)' \\ C^{\$}_{n-1,0}(3) - \Gamma_0(3) + (\theta-1)\kappa_1 A_0(3) + \frac{1}{2}\Psi_{n-1,c}(3)\Psi_{n-1,c}(3)' \end{bmatrix} + \begin{bmatrix} \Xi_c(1) \\ \Xi_c(2) \\ \Xi_c(3) \end{bmatrix}
$$

$$
\Xi_c(S_t) = \theta \log \delta + (\theta - 1)(\kappa_0 - A_0(S_t)) - \gamma \mu_c + \frac{1}{2}\gamma^2 e_1 \Sigma \Sigma' e_1'
$$

$$
\Psi_{n-1,c}(S_t) = \left\{C^{\$}_{n-1,1}(S_t) + (\theta - 1)\kappa_1 A_1(S_t) - \Gamma_1(S_t)\right\} \Omega(S_t) \Sigma_x(S_t).
$$

The loadings on nominal bond yields are

$$
B^{\$}_{n,0} = -\frac{1}{n} C^{\$}_{n,0}, \quad B^{\$}_{n,1} = -\frac{1}{n} C^{\$}_{n,1}.
$$

The log return to holding a n-maturity nominal bond from t to $t+1$ is

$$
\begin{aligned}
r^{\$}_{n,t+1} &= C^{\$}_{n-1,0}(S_{t+1}) - C^{\$}_{n,0}(S_t) + \left(C^{\$}_{n-1,1}(S_{t+1})\Phi(S_{t+1}) - C^{\$}_{n,1}(S_t)\right) X_t \\
&\quad + C^{\$}_{n-1,1}(S_{t+1}) \Omega(S_{t+1}) \Sigma_x(S_{t+1}) \eta_{x,t+1}.
\end{aligned}
$$

The log return to holding a n-maturity nominal bond from t to $t+1$ in excess of the log return to a one-period nominal bond is

$$
\begin{aligned}
rx^{\$}_{n,t+1} &= C^{\$}_{n-1,0}(S_{t+1}) - C^{\$}_{n,0}(S_t) + C^{\$}_{1,0}(S_t) + \left(C^{\$}_{n-1,1}(S_{t+1})\Phi(S_{t+1}) - C^{\$}_{n,1}(S_t) + C^{\$}_{1,1}(S_t)\right) X_t \\
&\quad + C^{\$}_{n-1,1}(S_{t+1}) \Omega(S_{t+1}) \Sigma_x(S_{t+1}) \eta_{x,t+1}.
\end{aligned}
$$

The one-period expected excess return on nominal bonds can be written in the following form:

$$E(rx^{\$}_{n,t+1}|S_t=k)+\frac{1}{2}Var(rx^{\$}_{n,t+1}|S_t=k)=-Cov(m^{\$}_{t+1},rx^{\$}_{n,t+1}|S_t=k)$$

$$=-\Pi(k,:)\times\left(\begin{bmatrix}\Lambda_1(k,1)\Lambda_2(k,1)+\Lambda_3(k,1)\\ \Lambda_1(k,2)\Lambda_2(k,2)+\Lambda_3(k,2)\\ \Lambda_1(k,3)\Lambda_2(k,3)+\Lambda_3(k,3)\end{bmatrix}\right)$$

$$\approx-\Pi(k,:)\times\begin{bmatrix}\Lambda_3(k,1)\\ \Lambda_3(k,2)\\ \Lambda_3(k,3)\end{bmatrix},$$

where the approximation is exact in a fixed-regime economy and

$$\Lambda_1(k,j)=K_t'C(j)'-\Pi(k,:)\begin{bmatrix}K_t'C(1)'\\ K_t'C(2)'\\ K_t'C(3)'\end{bmatrix}$$

$$\Lambda_2(k,j)=M(j)K_t-\Pi(k,:)\begin{bmatrix}M(1)K_t\\ M(2)K_t\\ M(3)K_t\end{bmatrix}$$

$$\Lambda_3(k,j)=\Big((\theta-1)\kappa_1A_1(j)-\Gamma_1(j)\Big)\Omega(j)\Sigma_x(j)\Sigma_x(j)'\Omega_x(j)'(C^{\$}_{n-1,1}(j))'$$

and

$$M(j)=\big[(\theta-1)\kappa_1A_0(j)-\Gamma_0(j)\qquad(\theta-1)\kappa_1A_1(j)\Phi(j)-\Gamma_1(j)\Phi(j)\big]$$

$$C(j)=\Big[C^{\$}_{n-1,0}(j)\qquad C^{\$}_{n-1,1}(j)\Phi(j)\Big]$$

$$K_t=\begin{bmatrix}1\\ X_t\end{bmatrix}.$$

Intuitively, I consider the limiting case in which the regime is fixed and $\rho_\pi=1$. The one-period expected excess return is $E_t(rx^{\$}_{n,t+1})+\frac{1}{2}Var_t(rx^{\$}_{n,t+1})$

$$=-\big((\theta-1)\kappa_1A_1-\Gamma_1\big)\Omega\Sigma_x\Sigma_x'\Omega_x'(C^{\$}_{n-1,1})'$$

$$=-(n-1)\sigma^2_{xc}\left\{(\gamma-\frac{1}{\psi})\frac{\kappa_1}{1-\kappa_1\rho_c}+\frac{1/\psi-\tau_c}{\tau_\pi-\rho_c}\right\}$$

$$\left\{\left(\frac{1/\psi\tau_\pi-\rho_c\tau_c}{\tau_\pi-\rho_c}\right)\frac{1}{n-1}\left(\frac{1-\rho_c^{n-1}}{1-\rho_c}\right)+\beta\right\}$$

$$-(n-1)\sigma^2_{xc}\left\{\left(\frac{1/\psi\tau_\pi-\rho_c\tau_c}{\tau_\pi-\rho_c}\right)\frac{1}{n-1}\left(\frac{1-\rho_c^{n-1}}{1-\rho_c}\right)\right\}\beta-(n-1)\sigma^2_{xc}\left(\beta^2+\frac{\sigma^2_{x\pi}}{\sigma^2_{xc}}\right)$$

where

$$-\big((\theta-1)\kappa_1A_1-\Gamma_1\big)=\Big[(\gamma-\tfrac{1}{\psi})\tfrac{\kappa_1}{1-\kappa_1\rho_c}+\tfrac{1/\psi-\tau_c}{\tau_\pi-\rho_c},\ 1\Big]$$

$$\Omega\Sigma_x\Sigma_x'\Omega_x'=\begin{bmatrix}\sigma^2_{xc} & \beta\sigma^2_{xc}\\ \beta\sigma^2_{xc} & \beta^2\sigma^2_{xc}+\sigma^2_{x\pi}\end{bmatrix}$$

$$C^{\$}_{n-1,1}=\Big[-\Big(\tfrac{1/\psi\tau_\pi-\rho_c\tau_c}{\tau_\pi-\rho_c}\Big)\Big(\tfrac{1-\rho_c^{n-1}}{1-\rho_c}\Big),\ -(n-1)\Big].$$

After some tedious algebra, the sign of the one-period expected excess return can be expressed as

$$sign\left(E_t(rx^{\$}_{n,t+1}) + \frac{1}{2} Var_t(rx^{\$}_{n,t+1}) \right) = -sign\left[(B^{\$}_{n-1,1,c} + \beta) \left\{ \frac{(\gamma - 1/\psi)\kappa_1}{1 - \kappa_1 \rho_c} + \frac{1/\psi - \tau_c}{\tau_\pi - \rho_c} + \beta \right\} + \frac{\sigma^2_{x\pi}}{\sigma^2_{xc}} \right].$$

C.10 Real Stock and Nominal Bond Return Correlation

The return on dividend claim (i.e., market return) is

$$r_{m,t+1} = \kappa_{0,m} + \mu_d + \kappa_{1,m} A_{0,m}(S_{t+1}) - A_{0,m}(S_t) + \left(\phi e_1 + \kappa_{1,m} A_{1,m}(S_{t+1}) \Phi(S_{t+1}) - A_{1,m}(S_t) \right) X_t$$

$$+ \kappa_{1,m} A_{1,m}(S_{t+1}) \Omega(S_{t+1}) \Sigma_x(S_{t+1}) \eta_{x,t+1} + e_2 \Sigma \eta_{t+1}.$$

The conditional covariance between the two is

$$Cov(r^{\$}_{n,t+1}, r_{m,t+1} | S_t = k) = \Pi(k,:) \times \left(\begin{bmatrix} \Lambda_1(k,1)\Lambda_2(k,1) + \Lambda_3(k,1) \\ \Lambda_1(k,2)\Lambda_2(k,2) + \Lambda_3(k,2) \\ \Lambda_1(k,3)\Lambda_2(k,3) + \Lambda_3(k,3) \end{bmatrix} \right)$$

$$\approx \Pi(k,:) \times \begin{bmatrix} \Lambda_3(k,1) \\ \Lambda_3(k,2) \\ \Lambda_3(k,3) \end{bmatrix}$$

where the approximation is exact in a fixed-regime economy and

$$\Lambda_1(k,j) = K_t' C(j)' - \Pi(k,:) \begin{bmatrix} K_t' C(1)' \\ K_t' C(2)' \\ K_t' C(3)' \end{bmatrix}$$

$$\Lambda_2(k,j) = A_m(j) K_t - \Pi(k,:) \begin{bmatrix} A_m(1) K_t \\ A_m(2) K_t \\ A_m(3) K_t \end{bmatrix}$$

$$\Lambda_3(k,j) = \kappa_{1,m} A_{1,m}(j) \Omega(j) \Sigma_x(j) \Sigma_x(j)' \Omega_x(j)' (C^{\$}_{n-1,1}(j))'$$

and

$$A_m(j) = \left[\kappa_{1,m} A_{0,m}(j) \quad \kappa_{1,m} A_{0,m}(j) \Phi(j) \right]$$

$$C(j) = \left[C^{\$}_{n-1,0}(j) \quad C^{\$}_{n-1,1}(j) \Phi(j) \right]$$

$$K_t = \begin{bmatrix} 1 \\ X_t \end{bmatrix}.$$

I consider the limiting case in which the regime is fixed. The stock-bond return covariance is

$$Cov_t(r^{\$}_{n,t+1}, r_{m,t+1}) = -\kappa_{1,m} \sigma^2_{xc} \left(\frac{\phi - 1/\psi}{1 - \kappa_{1,m} \rho_c} \right) (n-1) \left(B^{\$}_{n-1,1,c} + \beta \right),$$

from which I can deduce that

$$sign(Cov_t(r^{\$}_{n,t+1}, r_{m,t+1})) = -sign\left(B^{\$}_{n-1,1,c} + \beta \right).$$

Since $rx^{\$}_{n,t+1} = r^{\$}_{n,t+1} - y^{\$}_{1,t}$,

$$sign(Cov_t(rx^{\$}_{n,t+1}, r_{m,t+1})) = -sign\left(B^{\$}_{n-1,1,c} + \beta \right).$$

C.11 k Step ahead Expectations

Any variable K_{t+1} that can be expressed as

$$K_{t+1}=\Lambda_0(S_{t+1})+\Lambda_1(S_{t+1})X_{t+1}$$

$$X_{t+1}=\Phi(S_{t+1})X_t+\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1}, \quad \eta_{x,t}\sim N(0,I)$$

has the following k-step-ahead expectation form:

$$E(K_{t+k}|S_t)=\underbrace{E(\Lambda_0(S_{t+k})|S_t)}_{\Lambda_0^{(k)}}+\underbrace{E(\Lambda_1(S_{t+k})\Phi(S_{t+k})\Phi(S_{t+k-1})\dots\Phi(S_{t+1})|S_t)}_{\Lambda_1^{(k)}}X_t \tag{C10}$$

$$\begin{aligned}K_{t+k}=&\Lambda_0(S_{t+k})+\Lambda_1(S_{t+k})\Phi(S_{t+k})\Phi(S_{t+k-1})\dots\Phi(S_{t+1})X_t\\&+\Lambda_1(S_{t+k})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+k}\\&+\sum_{i=0}^{k-2}\Lambda_1(S_{t+k})\prod_{j=0}^{i}\Phi(S_{t+k-j})\Omega(S_{t+k-i-1})\Sigma_x(S_{t+k-i-1})\eta_{x,t+k-i-1}.\end{aligned}$$

We can characterize the constant and the slope coefficients as

$$\Lambda_0^{(k)}(j)=\begin{bmatrix}\Lambda_0(1) & \Lambda_0(2) & \Lambda_0(3)\end{bmatrix}\begin{bmatrix}p_{11} & p_{21} & p_{31}\\ p_{12} & p_{22} & p_{32}\\ p_{13} & p_{23} & p_{33}\end{bmatrix}^{(k-1)}\times\begin{bmatrix}p_{j1}\\ p_{j2}\\ p_{j3}\end{bmatrix},$$

$$\begin{aligned}\Lambda_1^{(k)}(j)=&\begin{bmatrix}\Lambda_1(1) & \Lambda_1(2) & \Lambda_1(3)\end{bmatrix}\begin{bmatrix}p_{11}\Phi(1) & p_{21}\Phi(1) & p_{31}\Phi(1)\\ p_{12}\Phi(2) & p_{22}\Phi(2) & p_{32}\Phi(2)\\ p_{13}\Phi(3) & p_{23}\Phi(3) & p_{33}\Phi(3)\end{bmatrix}^{(k-1)}\\&\times\begin{bmatrix}\Phi(1) & 0 & 0\\ 0 & \Phi(2) & 0\\ 0 & 0 & \Phi(3)\end{bmatrix}\begin{bmatrix}p_{j1}\mathbf{I}\\ p_{j2}\mathbf{I}\\ p_{j3}\mathbf{I}\end{bmatrix}.\end{aligned}$$

The cumulative k-step-ahead expectation is

$$\begin{aligned}\sum_{i=0}^{k-1}E(K_{t+i}|S_t=j)=&\left(\Lambda_0^{(0)}(j)+\Lambda_0^{(1)}(j)+\dots+\Lambda_0^{(k-1)}(j)\right)\\&+\left(\Lambda_1^{(0)}(j)+\Lambda_1^{(1)}(j)+\dots+\Lambda_1^{(k-1)}(j)\right)X_t.\end{aligned} \tag{C11}$$

C.12 Yield Decomposition: Average Expected Short Rates and Term Premium

The log yield for n-maturity nominal bond can be expressed as

$$y_{n,t}^{\$}=-\frac{1}{n}C_{n,0}^{\$}(S_t)-\frac{1}{n}C_{n,1}^{\$}(S_t)X_t.$$

Define the term premium for n-maturity bond by $\xi^{\$}_{n,t}$

$$y^{\$}_{n,t} = \frac{1}{n} E_t \sum_{i=0}^{n-1} y^{\$}_{1,t+i} + \xi^{\$}_{n,t}. \tag{C12}$$

We can compute

$$\begin{aligned} \xi^{\$}_{n,t} &= y^{\$}_{n,t} - \frac{1}{n} \sum_{i=0}^{n-1} E(y^{\$}_{1,t+i} | S_t = j), \\ &= \frac{1}{n}\left(C^{\$,(0)}_{1,0}(j) + \ldots + C^{\$,(n-1)}_{1,0}(j) - C^{\$}_{n,0}(j) \right) + \frac{1}{n}\left(C^{\$,(0)}_{1,1}(j) + \ldots + C^{\$,(n-1)}_{1,1}(j) - C^{\$}_{n,1}(j) \right) X_t. \end{aligned} \tag{C13}$$

C.13 Relationship between Term Premium and Risk Premium

Note that

$$\begin{aligned} y^{\$}_{n,t} &= -\frac{1}{n} p^{\$}_{n,t} \\ &= \frac{1}{n}\Bigg(\underbrace{-p^{\$}_{n,t} + p^{\$}_{n-1,t+1}}_{r^{\$}_{n,t+1}} \underbrace{-p^{\$}_{n-1,t+1} + p^{\$}_{n-2,t+2}}_{r^{\$}_{n-1,t+2}} - p^{\$}_{n-2,t+2} + \cdots \underbrace{- p^{\$}_{2,t+n-2} + p^{\$}_{1,t+n-1}}_{r^{\$}_{2,t+n-1}} \underbrace{- p^{\$}_{1,t+n-1}}_{y^{\$}_{1,t+n-1}} \Bigg). \end{aligned} \tag{C14}$$

Take the expectation and express the identity ex-ante as

$$y^{\$}_{n,t} = \frac{1}{n} E_t \left(r^{\$}_{n,t+1} + r^{\$}_{n-1,t+2} + \cdots + r^{\$}_{2,t+n-1} + y^{\$}_{1,t+n-1} \right). \tag{C15}$$

Since expected return can be decomposed into risk-free rate plus term premium, we re-express as

$$\begin{aligned} y^{\$}_{n,t} &= \frac{1}{n} E_t \left(rx^{\$}_{n,t+1} + rx^{\$}_{n-1,t+2} + \cdots + rx^{\$}_{2,t+n-1} \right) \\ &\quad \underbrace{+ \frac{1}{n} E_t \left(y^{\$}_{1,t} + y^{\$}_{1,t+1} + \cdots + y^{\$}_{1,t+n-2} + y^{\$}_{1,t+n-1} \right).}_{\frac{1}{n} E_t \sum_{i=0}^{n-1} y^{\$}_{1,t+i}} \end{aligned} \tag{C16}$$

Combine the expression in (C12) with (C16), and we have the following identity:

$$\begin{aligned} \xi^{\$}_{n,t} &= \frac{1}{n} E_t \left(rx^{\$}_{n,t+1} + rx^{\$}_{n-1,t+2} + \cdots + rx^{\$}_{2,t+n-1} \right) \\ &= \frac{1}{n} E_t rx^{\$}_{n,t+1} + \frac{1}{n} \sum_{j=1}^{n-2} E_t rx^{\$}_{n-j,t+1+j}. \end{aligned} \tag{C17}$$

Therefore, term premium is the average of expected future risk premiums of declining maturity.

C.14 News and Shock Decomposition

For illustrative purposes, I assume that $S_{t-1}=m$ and $S_t=j$.

$$\eta_{K,t}^{(k)}=E_t\left(\frac{1}{k}\sum_{i=1}^{k}K_{t+i}\right)-E_{t-1}\left(\frac{1}{k}\sum_{i=1}^{k}K_{t+i}\right) \tag{C18}$$

$$=E\left(\frac{1}{k}\sum_{i=1}^{k}K_{t+i}|S_t=j\right)-E\left(\frac{1}{k}\sum_{i=1}^{k}K_{t+i}|S_{t-1}=m\right)$$

$$=\frac{1}{k}\left(\Lambda_0^{(1)}(j)+\ldots+\Lambda_0^{(k)}(j)\right)+\frac{1}{k}\left(\Lambda_1^{(1)}(j)+\ldots+\Lambda_1^{(k)}(j)\right)X_t$$

$$-\frac{1}{k}\left(\Lambda_0^{(2)}(m)+\ldots+\Lambda_0^{(k+1)}(m)\right)-\frac{1}{k}\left(\Lambda_1^{(2)}(m)+\ldots+\Lambda_1^{(k+1)}(m)\right)X_{t-1}$$

$$=\frac{1}{k}\left(\Lambda_0^{(1)}(j)+\ldots+\Lambda_0^{(k)}(j)\right)+\frac{1}{k}\left(\Lambda_1^{(1)}(j)+\ldots+\Lambda_1^{(k)}(j)\right)\left(\Phi(j)X_{t-1}+\Omega(j)\Sigma_x(j)\eta_{x,t}\right)$$

$$-\frac{1}{k}\left(\Lambda_0^{(2)}(m)+\ldots+\Lambda_0^{(k+1)}(m)\right)-\frac{1}{k}\left(\Lambda_1^{(2)}(m)+\ldots+\Lambda_1^{(k+1)}(m)\right)X_{t-1}$$

$$=\frac{1}{k}\left(\Lambda_0^{(1)}(j)+\ldots+\Lambda_0^{(k)}(j)\right)-\frac{1}{k}\left(\Lambda_0^{(2)}(m)+\ldots+\Lambda_0^{(k+1)}(m)\right)$$

$$+\frac{1}{k}\left(\Lambda_1^{(1)}(j)+\ldots+\Lambda_1^{(k)}(j)\right)\Phi(j)X_{t-1}-\frac{1}{k}\left(\Lambda_1^{(2)}(m)+\ldots+\Lambda_1^{(k+1)}(m)\right)X_{t-1}$$

$$+\frac{1}{k}\left(\Lambda_1^{(1)}(j)+\ldots+\Lambda_1^{(k)}(j)\right)\Omega(j)\Sigma_x(j)\eta_{x,t}$$

$$=\frac{1}{k}\left(\left\{\Lambda_0^{(1)}(j)-\Lambda_0^{(2)}(m)\right\}+\ldots+\left\{\Lambda_0^{(k)}(j)-\Lambda_0^{(k+1)}(m)\right\}\right)$$

$$+\frac{1}{k}\left(\left\{\Lambda_1^{(1)}(j)\Phi(j)-\Lambda_1^{(2)}(m)\right\}+\ldots+\left\{\Lambda_1^{(k)}(j)\Phi(j)-\Lambda_1^{(k+1)}(m)\right\}\right)X_{t-1}$$

$$+\frac{1}{k}\left(\Lambda_1^{(1)}(j)+\ldots+\Lambda_1^{(k)}(j)\right)\Omega(j)\Sigma_x(j)\eta_{x,t}$$

represents the news.

$$\varepsilon_{K,t}=K_t-E\left(K_t|S_{t-1}\right) \tag{C19}$$

$$=\Lambda_0(j)+\Lambda_1(j)X_t-\Lambda_0^{(1)}(m)-\Lambda_1^{(1)}(m)X_{t-1}$$

$$=\Lambda_0(j)+\Lambda_1(j)\left(\Phi(j)X_{t-1}+\Omega(j)\Sigma_x(j)\eta_{x,t}\right)-\Lambda_0^{(1)}(m)-\Lambda_1^{(1)}(m)X_{t-1}$$

$$=\left(\Lambda_0(j)-\Lambda_0^{(1)}(m)\right)+\left(\Lambda_1(j)\Phi(j)-\Lambda_1^{(1)}(m)\right)X_{t-1}+\Lambda_1(j)\Omega(j)\Sigma_x(j)\eta_{x,t}$$

represents the shocks.

If $S_{t-1}=j$ and $S_t=j$, we can express $\eta_{K,t}^{(k)}$ and $\varepsilon_{K,t}$ by

$$\eta_{K,t}^{(k)}=\frac{1}{k}\left(\left\{\Lambda_0^{(1)}(j)-\Lambda_0^{(2)}(j)\right\}+\ldots+\left\{\Lambda_0^{(k)}(j)-\Lambda_0^{(k+1)}(j)\right\}\right) \tag{C20}$$

$$+\frac{1}{k}\left(\left\{\Lambda_1^{(1)}(j)\Phi(j)-\Lambda_1^{(2)}(j)\right\}+\ldots+\left\{\Lambda_1^{(k)}(j)\Phi(j)-\Lambda_1^{(k+1)}(j)\right\}\right)X_{t-1}$$

$$+\frac{1}{k}\left(\Lambda_1^{(1)}(j)+\ldots+\Lambda_1^{(k)}(j)\right)\Omega(j)\Sigma_x(j)\eta_{x,t}$$

$$\approx\frac{1}{k}\left(\Lambda_1^{(1)}(j)+\ldots+\Lambda_1^{(k)}(j)\right)\Omega(j)\Sigma_x(j)\eta_{x,t},$$

$$\varepsilon_{K,t}=\left(\Lambda_0(j)-\Lambda_0^{(1)}(j)\right)+\left(\Lambda_1(j)\Phi(j)-\Lambda_1^{(1)}(j)\right)X_{t-1}+\Lambda_1(j)\Omega(j)\Sigma_x(j)\eta_{x,t}$$

$$\approx\Lambda_1(j)\Omega(j)\Sigma_x(j)\eta_{x,t}.$$

Appendix D. Bayesian Inference

Posterior inference is implemented with a Metropolis-within-Gibbs sampler, which builds on the work of Carter and Kohn (1994); Kim and Nelson (1999). $Y_{1:T}$ denotes the sequence of observations, where

$$Y_t=(\Delta c_t,\pi_t,pd_t,y_{1,t},y_{2,t},y_{3,t},y_{4,t},y_{5,t}).$$

Moreover, let $S_{1:T}$ be the sequence of hidden states, and let $\Theta=(\Theta_1,\Theta_2)$, where

$$\Theta_1=(\delta,\gamma,\psi),$$

$$\Theta_2=(\mu_c,\mu_d,\rho_c,\rho_i,\phi,\sigma_c,\sigma_d,\sigma_{xc},\sigma_{x\pi},\sigma_{xi},\beta(-),\beta(+),\tau_0(P),\tau_0(A),\tau_c(P),\tau_c(A),\tau_\pi(P),\tau_\pi(A)),$$

$$\Phi=(\{\Pi_{ij}\}_{i,j=\{1,2,3\}}).$$

The Metropolis-within-Gibbs algorithm involves iteratively sampling from three conditional posterior distributions. To initialize the sampler, I start from (Θ^0,Φ^0).

D.1 Algorithm: Metropolis Sampler

For $i=1,\ldots,N$:

1. Draw $S_{1:T}^{i+1}$ conditional on Θ^i, Φ^i, and $Y_{1:T}$. This step is implemented using the multi-move simulation smoother described in Section 9.1.1 of Kim and Nelson (1999).
2. Draw Φ^{i+1} conditional on Θ^i, $S_{1:T}^{i+1}$, and $Y_{1:T}$. If the dependence of the distribution of the initial state S_1 on Φ is ignored, then it can be shown that the conditional posterior of Φ is of the Dirichlet form. I use the resultant Dirichlet distribution as a proposal distribution in a Metropolis-Hastings step.
3. Draw Θ^{i+1}, conditional on Φ^{i+1}, $S_{1:T}^{i+1}$, and $Y_{1:T}$. Since the prior distribution is nonconjugate, I am using a random-walk Metropolis step to generate a draw from the conditional posterior of Θ. The proposal distribution is $N(\Theta^i,c\Omega)$.

I obtain the covariance matrix Ω of the proposal distribution in Step 2 as follows. Following Schorfheide (2005), I maximize the posterior density,

$$p(\Theta|Y_{1:T})\propto p(Y_{1:T}|\Theta)p(\Theta),$$

to obtain the posterior mode $\tilde{\Theta}$. I then construct the negative inverse of the Hessian at the mode. I choose the scaling factor c to obtain an acceptance rate of approximately 40 percent. We initialize our algorithm choosing (Θ^0) in the neighborhood of $(\tilde{\Theta})$ and use it to generate $N=100,000$ draws from the posterior distribution.

Table D-4
Prior distributions

	Distr.	5%	50%	95%		Distr.	5%	50%	95%
		Factor shocks					Transition probability		
$\beta(+)$	U	0.5	5.0	9.5	p_{11}	Dir	0.85	0.90	0.95
$\beta(-)$	U	−9.5	−5.0	−0.5	p_{21}	Dir	0.02	0.05	0.09
σ	IG	0.0008	0.0019	0.0061	p_{31}	Dir	0.02	0.05	0.09
		Monetary policy			p_{12}	Dir	0.02	0.05	0.09
τ_0	U	0.0014	0.0055	0.0095	p_{22}	Dir	0.85	0.90	0.95
$\tau_\pi(A)$	U	1.10	2.00	2.90	p_{32}	Dir	0.02	0.05	0.09
$\tau_\pi(P)$	U	0.05	0.50	0.95	p_{13}	Dir	0.02	0.05	0.09
τ_c	U	0.05	0.50	0.95	p_{23}	Dir	0.85	0.90	0.95
					p_{33}	Dir	0.02	0.05	0.09

Notes: Dir, IG, and U denote the dirichlet, inverse gamma, and uniform distributions, respectively.

Appendix E. Supplementary Figures and Tables

E.15 Model-Implied Moments: Real Bond Market

The one-period expected excess return on a n-maturity real bond is

$$E_t(rx_{n,t+1})+\frac{1}{2}Var_t(rx_{n,t+1})=-(\gamma-\frac{1}{\psi})\left(\frac{\kappa_1/\psi}{1-\kappa_1\rho_c}\right)\left(\frac{1-\rho_c^{n-1}}{1-\rho_c}\right) \tag{E21}$$

in the absence of regime switching (derivation is provided in (C6)). When agents have a preference for an early resolution of uncertainty ($\gamma > 1/\psi$), real bonds are hedges against low growth and real bond risk premiums are always negative. Because these hedging effects are stronger at longer horizons, the model-implied real term structure is downward sloping. See Table E-5.

Table E-5
Model-implied moments: Real bond

	Data	Regime-switching model			
		Average real bond yield			
Maturity		Mix	CA	CP	PA
1-year bond	–	1.64	1.60	1.59	1.64
2-year bond	0.19	1.60	1.55	1.54	1.60
3-year bond	0.33	1.53	1.50	1.50	1.53
4-year bond	0.49	1.50	1.46	1.47	1.50
5-year bond	1.38	1.46	1.41	1.41	1.46
		Real bond risk premiums			
1-year bond	–	−0.08	−0.03	−0.03	−0.03
2-year bond	–	−0.16	−0.05	−0.06	−0.05
3-year bond	–	−0.24	−0.08	−0.08	−0.08
4-year bond	–	−0.30	−0.10	−0.11	−0.10
5-year bond	–	−0.36	−0.11	−0.13	−0.12

Notes: The model-implied average real bond yields and risk premiums are provided. Treasury Inflation-Indexed Security bonds are obtained from the Board of Governors of the Federal Reserve System. The TIPS yields with maturities at two to four years are available from 2004 to 2015. The TIPS yields with maturities greater than or equal to five years are available from 1999.

E.16 Inflation Forecasts from the Survey of Professional Forecasters

A rich data set of individual CPI inflation survey forecasts taken from the Survey of Professional Forecasters is available at the Federal Reserve Bank of Philadelphia. The shares of each response group, categorized into $\pi<0, 0\leq\pi<1, 1\leq\pi<2, 2\leq\pi<3, 3\leq\pi<4$, and $4\leq\pi$, for the 1-quarter-ahead inflation forecasts (light-blue-squared) and 1-year-ahead inflation forecasts (black-solid) are provided in Figure E-4. The mean of the 1-quarter-ahead inflation forecasts (light-blue-squared) and 1-year-ahead inflation forecasts (black-solid) are approximately 2.3%. It is interesting to observe that the number of survey respondents who forecast inflation to be greater than the average value is very large. For example, more than half of the survey respondents predicted inflation to be greater than 3% during the periods between 2005 and 2009. In particular, around half of the respondents predicted inflation to exceed 4% during the Great Recession.

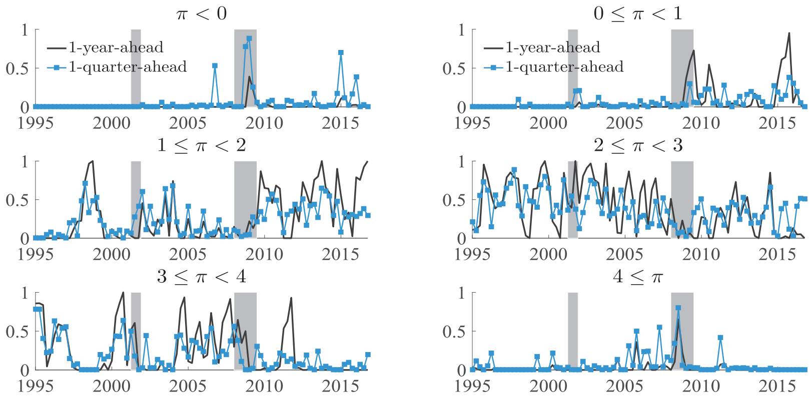

Figure E-4
Inflation forecasts from the Survey of Professional Forecasters
CPI inflation forecasts (quarter-over-quarter, annualized percentage points) for all individual forecasters are available at the Federal Reserve Bank of Philadelphia. I categorize individual responses into 6 groups; $\pi<0$, $0\leq\pi<1$, $1\leq\pi<2$, $2\leq\pi<3$, $3\leq\pi<4$, and $4\leq\pi$. The figure reports the share of each response group for the 1-year-ahead inflation forecasts (black-solid) and 1-quarter-ahead inflation forecasts (light-blue-squared).

E.17 Campbell, Pflueger, and Viceira (2015) Revisited

I estimate the New Keynesian model proposed by Campbell, Pflueger, and Viceira (2015). The model has three structural equations (IS curve, Phillips curve, and Monetary policy rule) and uses the law of motion for time-varying inflation target.

1. IS curve : $x_t = \rho^{x-}(S_t)x_{t-1} + \rho^{x+}(S_t)E_t x_{t+1} - \frac{\psi(S_t)}{4}(i_t - E_t\pi_{t+1}) + u_t^{IS}$
2. Phillips curve : $\pi_t = \rho^{\pi}(S_t)\pi_{t-1} + (1-\rho^{\pi}(S_t))E_t\pi_{t+1} + 4\kappa(S_t)x_t + 4u_t^{PC}$
3. Monetary policy : $i_t = \rho^i(S_t)i_{t-1} + (1-\rho^i(S_t))\big(4\gamma^x(S_t)x_t + \gamma^{\pi}(S_t)(\pi_t - \pi_t^{TG}) + \pi_t^{TG}\big) + 4u_t^{MP}$
4. Inflation target : $\pi_t^{TG} = \pi_{t-1}^{TG} + 4u_t^{TG}$,

where x_t is the log output gap, π_t is the inflation rate, i_t is the log yield of a one month maturity at time t, π_t^{TG} is inflation target, and $u_t = [u_t^{IS}, u_t^{PC}, u_t^{MP}, u_t^{TG}]'$, $E_{t-1}[u_t u_t'] = \Sigma_u$ is diagonal and homoskedastic.

Table E-6
Posterior estimates: New Keynesian model

	Prior			Posterior								
				Passive			Active			Active(+)		
	Distr.	5%	95%	50%	5%	95%	50%	5%	95%	50%	5%	95%
						Preferences						
ρ^{x-}	U	[0.03	0.47]	0.243	[0.184	0.340]	0.118	[0.070	0.145]	0.118	[0.057	0.133]
ρ^{x+}	U	[0.54	1.45]	0.899	[0.799	0.956]	0.899	[0.796	1.045]	0.900	[0.852	1.018]
ψ	U	[0.01	2.25]	0.038	[0.016	0.053]	0.003	[0.000	0.070]	0.006	[0.000	0.013]
ρ^{π}	B	[0.21	0.98]	0.850	[0.767	0.965]	0.750	[0.602	0.890]	0.887	[0.752	0.984]
κ	U	[0.03	0.47]	0.342	[0.239	0.395]	0.194	[0.110	0.239]	0.220	[0.025	0.263]
						Monetary policy rule coefficients						
ρ^{i}	B	[0.21	0.98]	0.531	[0.412	0.668]	0.980	[0.937	0.999]	0.912	[0.900	0.990]
γ^{π}	U	[0.01	2.25]	0.685	[0.376	0.952]	1.080	[1.000	1.109]	1.101	[1.000	1.140]
γ^{x}	U	[0.01	1.00]	0.971	[0.892	1.000]	0.999	[0.814	1.000]	0.999	[0.891	1.000]
						Standard deviations of macro shocks						
σ^{IS}	IG	[0.05	1.25]	0.971	[0.719	1.241]	0.226	[0.197	0.326]	0.476	[0.378	0.559]
σ^{PC}	IG	[0.05	1.25]	0.369	[0.319	0.441]	0.199	[0.127	0.301]	0.175	[0.118	0.259]
σ^{MP}	IG	[0.05	1.25]	0.383	[0.325	0.434]	0.007	[0.000	0.011]	0.107	[0.050	0.133]
σ^{TG}	IG	[0.05	1.25]	0.815	[0.668	0.902]	0.201	[0.148	0.256]	0.360	[0.247	0.438]

Notes: B, U, and IG denote the beta, uniform, and inverse gamma distributions, respectively.

I employ the solution algorithm proposed by Farmer, Waggoner, and Zha (2011) (minimal state variable solutions to Markov-switching rational expectations models)

$$\alpha_t = \left[x_t, E_t x_{t+1}, \pi_t, E_t \pi_{t+1}, i_t, \pi_t^{TG}\right]'$$

$$\eta_t = [x_t - E_{t-1} x_t, \pi_t - E_{t-1} \pi_t]$$

$$\Gamma_0(s_t)\alpha_t = \Gamma_1(s_t)\alpha_{t-1} + \Psi(s_t)u_t + \Pi(s_t)\eta_t$$

and cast the model into state-space representation

$$Y_t = \Lambda \alpha_t, \quad Y_t = [x_t, 4\pi_t, 4i_t]',$$

$$\alpha_t = \Phi(s_t)\alpha_{t-1} + \Sigma(s_t)u_t.$$

I use the Bayesian method (described in the previous section) to make inference on model coefficients and the regime probabilities

$$\Theta|(s_t = i) = \left\{\rho^{x-}, \rho^{x+}, \psi, \rho^{\pi}, \kappa, \rho^{i}, \gamma^{x}, \gamma^{\pi}, \Sigma_u\right\},$$

$$Pr(s_t = i), i \in \{\text{Passive, Active, Active(+)}\}.$$

The state space is estimated with output gap, inflation, and the federal funds rate (without asset data).

There are several takeaways from this exercise. First, the estimated monetary policy coefficients and the smoothed regime probabilities in Figure E-5 are broadly consistent with Table 2 and Figure 3 in that the central bank's response to inflation (and to the output gap) has been active since the mid-1980s. Second, the variances of the structural innovations were largest from the 1970s to the mid-1980s (passive regime) which are consistent with Table 2 (captured by larger inflation innovation variance). The results are also consistent with Bianchi (2012); Davig and Doh (2014). Third, data do not strongly support the existence of the third monetary policy regime. It suffices to have a single active regime in addition to the passive regime. Although, the zero lower bound

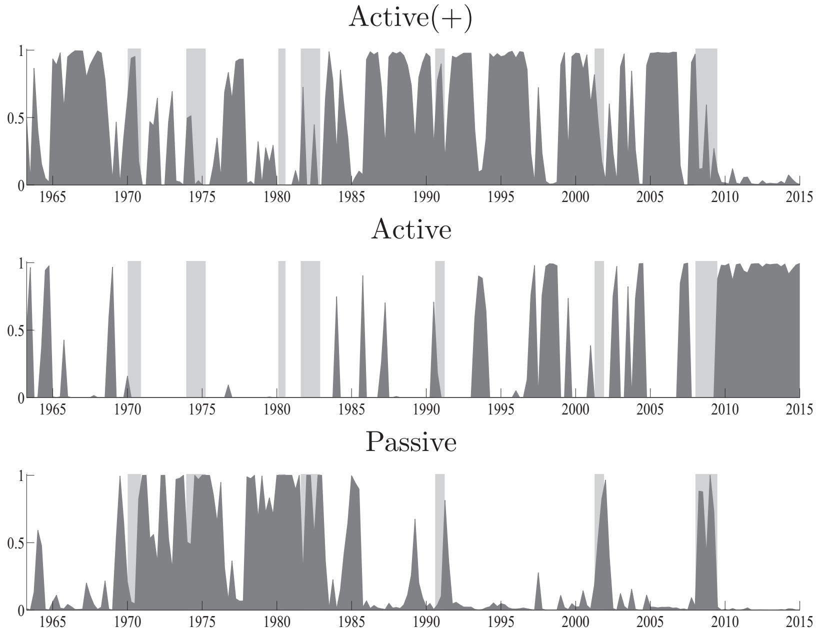

Figure E-5
Regime probabilities: New Keynesian model
The dark-gray-shaded areas represent posterior medians of regime probabilities. The light-gray-shaded bars indicate the NBER recession dates.

period is characterized by a separate active monetary policy regime that has policy rule inertia coefficient near one. This is expected because the rate has essentially been fixed at the zero lower bound. Fourth, and, more importantly, there is about a 1/3 probability of switching to the passive regime which corresponds to my CP regime. Overall, I find that the paths for monetary policy under the New Keynesian model are broadly consistent with my proposed model.

E.18 The Interpretation of β

It is important to think about the interpretation of β given its key role in the model specification. To answer it, let us take a detour and suppose that the innovation process to the long-run growth and to the inflation target can be decomposed into

$$\underbrace{\begin{bmatrix} \sigma_{xc} & 0 \\ 0 & \sigma_{x\pi} \end{bmatrix} \begin{bmatrix} \eta_{xc,t+1} \\ \eta_{x\pi,t+1} \end{bmatrix}}_{\text{Version I}} = \underbrace{\begin{bmatrix} \sigma^d_{xc} & \sigma^s_{xc} \\ \sigma^d_{x\pi} & -\sigma^s_{x\pi} \end{bmatrix} \begin{bmatrix} \eta_{xd,t+1} \\ \eta_{xs,t+1} \end{bmatrix}}_{\text{Version II}}, \tag{E22}$$

where $\eta_{xj,t} \sim N(0,1)$ for $j \in \{c, \pi, d, s\}$.[36] Note that $\eta_{xd,t}$ moves the long-run growth and inflation target in the same direction, hence can be interpreted as a demand shock. On the contrary, $\eta_{xs,t}$ moves the long-run growth and inflation target in the opposite direction, and thus be labeled as a

[36] "Version I" reproduces $\Sigma_x \eta_{x,t+1}$ in (3).

supply shock.[37] Then, the joint dynamics of the long-run growth and inflation target in (3) can be rewritten as

$$\begin{bmatrix} x_{c,t+1} \\ x_{\pi,t+1} \end{bmatrix} - \begin{bmatrix} \rho_c & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{c,t} \\ x_{\pi,t} \end{bmatrix} = \begin{bmatrix} \sigma^d_{xc} & \sigma^s_{xc} \\ \beta\sigma^d_{xc} + \sigma^d_{x\pi} & \beta\sigma^s_{xc} - \sigma^s_{x\pi} \end{bmatrix} \begin{bmatrix} \eta_{xd,t+1} \\ \eta_{xs,t+1} \end{bmatrix}. \tag{E23}$$

The economy in which the value of β is negative raises the relative importance of supply shocks and, importantly, translates adverse supply shocks (negative realizations of $\eta_{xs,t}$) into more persistent positive movements in the inflation target itself. In contrast, the positive value of β works to increase the relative role of demand shocks and decreases the impact of adverse supply shocks on the inflation target ($\beta\sigma^s_{xc}$ cancels out $-\sigma^s_{x\pi}$ to some extent). In sum, through regime switches in β, I attempt to capture such structural shifts in the economy in reduced form. For example, a flattening of the Phillips curve or an increased flexibility in wage dynamics could be potential candidates for structural shifts in the economy.[38]

A completely different view is that β could be a response function of the Federal Reserve's inflation targeting rule. Since the Federal Reserve has never explicitly revealed the setting for the inflation target, this view deserves a discussion. There is a very closely related paper that can shed light on this issue. Ireland (2007) estimates a New Keynesian model that allows the Federal Reserve's inflation target to respond systematically to supply shocks, for example, cost-push $\epsilon_{\theta,t}$ and technology $\epsilon_{z,t}$ shocks. Reproducing Equation (10) in Ireland (2007) below

$$x_{\pi,t} = x_{\pi,t-1} - \delta_\theta \epsilon_{\theta,t} - \delta_z \epsilon_{z,t} + \sigma_\pi \epsilon_{\pi,t}, \tag{E24}$$

where the response coefficients $\delta_\theta \geq 0$ and $\delta_z \geq 0$ are chosen by the Federal Reserve and $\epsilon_{\pi,t}$ is an idiosyncratic innovation to inflation target, $x_{\pi,t}$. Ireland (2007) finds that while δ_θ is estimated to enter significantly in the inflation target rule, the joint hypothesis that $\delta_\theta = 0$ and $\delta_z = 0$ cannot be rejected with any reasonable degree of confidence (using the likelihood-ratio test). Now, coming back to this paper, even if I decompose the model primitive shocks into (E22), the supply shock $\eta_{xs,t}$ is convolution of many structural shocks in the economy including the technology shock (which cannot be separately identified in this paper).[39] The empirical findings in Ireland (2007) suggest that inflation target does not significantly respond to the technology shock. Data are just inconclusive to support the view that policy parameters enter into the inflation target rule. There is still considerable uncertainty about the true source of shifts in the inflation target. Identifying the source of variations is beyond the scope of this paper.

To conclude the discussion, while it is entirely plausible to think otherwise, for this paper I am taking the view that β is not a policy parameter but rather captures exogenous shifts in the economy.

It is interesting to find out if the estimated regime probabilities under "Version II" in (E22) match those estimated under "Version I." "Version II" has a slightly more structural form than "Version I" in that the real growth and nominal inflation target innovations are decomposed into demand and supply shocks. The estimated regime probabilities under "Version I" versus "Version II" are nearly identical. The asset pricing implications of "Version II" are very similar to those from "Version I" reported in Table 8. The results are omitted for the sake of space.

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37 The decomposition is explored in Ermolov (2015); Bekaert, Engstrom, and Ermolov (2016).

38 Note that real marginal costs drive inflation in the basic Phillips curve and the real marginal costs are directly linked to wages. Among others, Blanchard and Gali (2009); Hofmann, Peersman, and Straub (2012) find that the effect of supply shocks on the economy has significantly reduced due to increased flexibility in wage dynamics.

39 Kaltenbrunner and Lochstoer (2010) argue that the innovation to the long-run growth or long-run risks is highly correlated with the technology shock.

Table E-7
Decompositions of variances of yield innovations: Expected inflation news

	Without measurement errors					
	Regime-switching model			Fixed-regime model		
Maturity	CA	CP	PA	CA	CP	PA
1-year bond	1.30	1.15	0.59	1.27	1.19	0.55
2-year bond	1.28	1.13	0.61	1.25	1.18	0.57
3-year bond	1.26	1.12	0.63	1.23	1.17	0.58
4-year bond	1.23	1.11	0.64	1.20	1.16	0.60
5-year bond	1.22	1.10	0.65	1.18	1.15	0.62
	With measurement errors					
1-year bond	0.42	0.52	0.13	0.46	0.63	0.11
2-year bond	0.42	0.53	0.14	0.46	0.63	0.11
3-year bond	0.42	0.53	0.14	0.46	0.62	0.11
4-year bond	0.41	0.53	0.14	0.45	0.62	0.11
5-year bond	0.41	0.54	0.14	0.45	0.61	0.11

Notes: The size of the measurement error variance is less than 5% of the sample variance.

E.19 Inflation Risks and Bond Yields

Duffee (2015) suggests that the role of news about expected future inflation in driving the variation of nominal yields has to be small for well-behaved term structure models. Duffee (2015) decomposes shocks to nominal bond yields $\epsilon_{y^{\$},n,t}$ into news about expected future inflation $\epsilon_{\pi,n,t}$, news about expected future real short rates $\epsilon_{y_1,n,t}$, and expected excess returns $\epsilon_{x,n,t}$. A yield shock is the sum of news

$$\epsilon_{y^{\$},n,t} = \epsilon_{\pi,n,t} + \epsilon_{y_1,n,t} + \epsilon_{x,n,t}, \tag{E25}$$

where

$$\epsilon_{y^{\$},n,t} = y^{\$}_{n,t} - E_{t-1}\left(y^{\$}_{n,t}\right) \tag{E26}$$

$$\epsilon_{\pi,n,t} = E_t\left(\frac{1}{n}\sum_{i=1}^{n}\pi_{t+i}\right) - E_{t-1}\left(\frac{1}{n}\sum_{i=1}^{n}\pi_{t+i}\right)$$

$$\epsilon_{y_1,n,t} = E_t\left(\frac{1}{n}\sum_{i=1}^{n}y_{1,t+i-1}\right) - E_{t-1}\left(\frac{1}{n}\sum_{i=1}^{n}y_{1,t+i-1}\right)$$

$$\epsilon_{x,n,t} = E_t\left(\frac{1}{n}\sum_{i=1}^{n}rx^{\$}_{n-i+1,t+i}\right) - E_{t-1}\left(\frac{1}{n}\sum_{i=1}^{n}rx^{\$}_{n-i+1,t+i}\right)$$

denotes the news.[40] The calculation of (E26) is explained in detail in Appendices C.11, C.12, and C.14.

Duffee (2015) defines a measure of inflation risk by

$$\text{inflation risk} = \frac{Var(\epsilon_{\pi,n,t})}{Var(\epsilon_{y^{\$},n,t})} \tag{E27}$$

and provides both survey- and model-based measures of (E27) that are estimated to be around 10% to 20%. He argues that this magnitude of inflation risk is strongly at odds with values implied by

40 The accounting identity lets us decompose the n-maturity nominal bond into future expected average inflation, real rates, and excess log returns: $y^{\$}_{n,t} = \frac{1}{n}\sum_{i=1}^{n} E_t(\pi_{t+i}) + \frac{1}{n}\sum_{i=1}^{n} E_t(y_{1,t+i-1}) + \frac{1}{n}\sum_{i=1}^{n} E_t(rx^{\$}_{n-i+1,t+i})$.

standard equilibrium models of inflation and bond yields. In particular, the standard term structure models (with recursive preferences and long-run risks) counterfactually imply too high of an inflation risk, which often exceeds one.

Table E-7 shows the model-implied inflation risks (E27), which exceed one in the CA and CP regimes and are less than one in the PA regime. The results are expected since inflation is countercyclical and risky in the CA and CP regimes. Based on the unconditional probabilities of regimes, I can compute the averages of inflation risks to be around one. This evidence speaks against the empirical validity of my model. However, if the measure of a yield shock, $\epsilon_{y^{\$},n,t} = y^{\$}_{n,t} - E_{t-1}\left(y^{\$}_{n,t}\right)$, includes measurement errors (whose variance is less than 5% of the sample variance), model-implied inflation risks (E27) become significantly smaller.[41]

Having said that, I can draw two important lessons from this exercise. First, it is important to relax the constant (time-invariant) parameter assumption since it overemphasizes the role of inflation risk in the yield curve. Second, allowing for regime switching is an economically appealing way of modeling inflation dynamics. Since each regime corresponds to a different level of inflation risk, a richer description of inflation dynamics is possible. As a result, it provides us a more comprehensive understanding of the sources of risk behind the yield curve.

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41 Bauer and Rudebusch (2015) and Cieslak and Povala (2015) show that the presence of small yield measurement errors can have a large impact on the spanning ability of interest rates and on the analysis of term premium, respectively.

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